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TOPICAL ISSUE

Finite-Frequency Object Identification with Delays

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Abstract—We propose a method of finite-frequency identification for stable objects with delay in the presence of an unknown external disturbance and measurement errors. This method uses a test signal which is the sum of at most as many harmonics as the number of coefficients in the object. In order to uniquely determine the delay value, we propose an approach that uses a test signal in the form of a product of a sinusoidal and exponential function. Based on this approach, we construct two identification algorithms: one for separate identification of coefficients and the delay value, another for their joint identification.

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1. INTRODUCTION

At present, control theory features many different methods for identifying objects defined with differential equations. These methods can be somewhat superficially divided into two categories depending on the assumptions regarding measurement errors and external disturbances that act upon the object.

Methods from the first category deal with objects that are subject to stochastic disturbances (e.g., white noise with known statistical characteristics). They represent different variations of the least squares method and stochastic approximation methods [1].

The second category includes methods that deal with unknown bounded external disturbances (whose statistical characteristics are unknown); this category includes randomized algorithms [2] and finite-frequency identification [3, 4].

The identification process may be either passive or active. In passive identification, the measured input of an object has the meaning of a controlling influence that depends on control objectives and is unrelated to object identification. Identification may be impossible for such an input. In these cases, one can try active identification that adds a signal to the input influence. Such a signal is called a *test signal*.

The finite-frequency identification method has been developed for active identification. The test signal is a sum of harmonics whose number does not increase the value of the object's state vector. Amplitudes and frequencies of this signal significantly influence identification accuracy, so one must choose them properly. In [5], the authors analyze the influence of frequencies and amplitudes of the test signal on identification accuracy and propose algorithms for tuning these frequencies and amplitudes. These algorithms ensure that the test signal influences the object's output in a very small way as compared to the influence of disturbances and noise.

One advantage of finite-frequency identification as compared to other existing methods is that despite unknown type and intensity of the external disturbance, it converges provided it does not contain components with test signal frequencies. In the case when an external influence or measurement noise do contain components with test signal frequencies, we can use the algorithm with successive pairs [6], where identification is done in two stages. On the first stage, we send a polyharmonic test signal to the object, and then the object receives the same test signal but with the opposite sign. Then we add identification results together and thus compensate the influence of external disturbances.

Besides, identification problems for objects with delay are also often considered. A survey of identification methods for such objects, mainly for random external disturbances and noises, is given in [7], where the authors also give recommendations on how to choose a suitable identification method. An adaptive observer able to estimate the object's coefficients and the value of delay is proposed in [8, 9]. In [10], two-stage identification is proposed. On the first stage, one uses a test signal which consists of a single harmonic or a sum of several harmonics. On the second stage, one uses identification with a ladder-like test influence.

In this work, we develop the finite-frequency identification method for objects with delay. Unlike [11], in this work we propose an approach that can uniquely determine the delay and construct the corresponding identification algorithms based on this approach. Tuning the frequencies and amplitudes of the test signal is in many ways similar to [5], and we do not consider it separately in this work.

The paper is organized as follows. In Section 2, we give the problem setting. Frequency identification equations with which we determine estimates on object coefficients are given in Section 3. In Section 4, we show an approach that can uniquely determine the delay. Conditions for identification convergence are given in Section 5. Sections 6 and 7 deal with identification algorithms, and Section 8 illustrates our results with an example.

2. PROBLEM SETTING

Consider a fully controllable asymptotically stable object with delay defined by equation

$$d_n y^{(n)}(t) + \cdots + d_1 \dot{y}(t) + y(t) = k_m u^{(m)}(t-\tau) + \cdots + k_0 u(t-\tau) + f(t), \tag{1}$$

where $y(t)$ is the object's output, $u(t)$ is the control input, τ is the delay ($\tau > 0$); the piecewise continuous function $f(t)$ is an unknown bounded disturbance:

$$|f(t)| \leqslant f^*,$$

where f^* is some unknown positive number.

Coefficients d_ν, k_μ ($\nu = \overline{1,n},\ \mu = \overline{0,m}$) and the delay τ are not known. Roots of the polynomial $k(s) = k_m s^m + k_{m-1}s^{m-1} + \cdots + k_1 s + k_0$ have negative real parts. Here and in what follows s denotes the Laplace transform.

The measured object's output has the form

$$\tilde{y}(t) = y(t) + \eta(t), \tag{2}$$

where $\eta(t)$ denotes measurement noise which is an unknown piecewise continuous bounded function.

The test signal is a sum of harmonics

$$u(t) = \sum_{i=1}^{l} \rho_i \sin \omega_i t, \tag{3}$$

where the number of harmonics equals $l = n + m + 1$; amplitudes ρ_i and frequencies ω_i are given positive numbers ($\omega_i \neq \omega_k,\ i \neq k,\ i = \overline{1,l},\ k = \overline{1,l}$).

Coefficients of object (1) often are piecewise constant functions. If the duration of the period when they are constant exceeds identification time, then the exposition below also relates to such objects.

The purpose of identification is to find estimates $\hat{\tau}$ of the delay and $\hat{d}_\nu$, $\hat{k}_\mu$ ($\nu = \overline{1,n}$, $\mu = \overline{0,m}$) of the coefficients for object (1) *such that the following conditions:*

$$\begin{cases} |\hat{d}_\nu - d_\nu| \leqslant \varepsilon_\nu^d |d_\nu|, \quad \text{if} \quad d_\nu \neq 0, \quad \text{or} \quad |\hat{d}_\nu| \leqslant \varepsilon_\nu^d, \quad \text{if} \quad d_\nu = 0 \\ |\hat{k}_\mu - k_\mu| \leqslant \varepsilon_\mu^k |k_\mu|, \quad \text{if} \quad k_\mu \neq 0, \quad \text{or} \quad |\hat{k}_\mu| \leqslant \varepsilon_\mu^k, \quad \text{if} \quad k_\mu = 0, \end{cases} \tag{4}$$

hold for the relative identification accuracy, where ε_ν^d and ε_μ^k ($\nu = \overline{1,n}, \mu = \overline{0,m}$) are given positive numbers.

3. IDENTIFICATION OF OBJECT COEFFICIENTS

To find object coefficients, we can get equations that are independent on the delay. To do so, we write the object's transition function (1) as

$$w_\tau(s) = w(s)e^{-\tau s} = \frac{k(s)}{d(s)} e^{-\tau s}, \tag{5}$$

where $d(s) = d_n s^n + d_{n-1} s^{n-1} + \cdots + d_1 s + 1$.

We introduce the numbers

$$\alpha_i = \mathrm{Re} w_\tau(j\omega_i) \quad \text{and} \quad \beta_i = \mathrm{Im} w_\tau(j\omega_i), \quad i = \overline{1,l}, \tag{6}$$

called *frequency parameters* of the control object (1).

Estimates of the object's frequency parameters are found empirically with a Fourier filter [3, 12]:

$$\hat{\alpha}_i = \alpha_i(\bar{t}) = \frac{2}{\rho_i \bar{t}} \int\limits_{t_F}^{t_F+\bar{t}} \tilde{y}(t) \sin \omega_i t \, dt \quad \text{and}$$
$$\hat{\beta}_i = \beta_i(\bar{t}) = \frac{2}{\rho_i \bar{t}} \int\limits_{t_F}^{t_F+\bar{t}} \tilde{y}(t) \cos \omega_i t \, dt, \quad i = \overline{1,l}, \tag{7}$$

where t_F is the time when filtering starts, and $\bar{t}$ is its duration.

Conditions regarding the convergence of filter outputs to true frequency parameters of object (6) are shown in Section 5.

The following equations relate frequency object parameters α_i and β_i ($i = \overline{1,l}$) with object coefficients k_μ ($\mu = \overline{1,m}$) and d_ν ($\nu = \overline{1,n}$):

$$k(j\omega_i)k(-j\omega_i) - (\alpha_i^2 + \beta_i^2)[d(j\omega_i)d(-j\omega_i) - 1] = \alpha_i^2 + \beta_i^2, \quad i = \overline{1,l}, \tag{8}$$

where

$$k(j\omega)k(-j\omega) = \sum_{\mu=0}^{m} (-1)^\mu \omega^{2\mu} \tilde{k}_\mu \quad \text{and} \quad d(j\omega)d(-j\omega) - 1 = \sum_{\nu=1}^{n} (-1)^\nu \omega^{2\nu} \tilde{d}_\nu \tag{9}$$

are even degree polynomials.

These equations are easy to get. To do so, we write (5) as

$$k(j\omega_i) = w_\tau(j\omega_i) e^{j\omega_i \tau} d(j\omega_i), \quad i = \overline{1,l},$$

and then multiply the left- and right-hand sides by the corresponding complex conjugate factors

$$k(j\omega_i)k(-j\omega_i) = w_\tau(j\omega_i)w_\tau(-j\omega_i)d(j\omega_i)d(-j\omega_i), \quad i = \overline{1,l}.$$

Using definition (6), we get (8).

Substituting (9) into (8), we get *frequency identification equations*

$$\sum_{\mu=0}^{m}(-1)^\mu \omega_i^{2\mu}\tilde{k}_\mu - (\alpha_i^2 + \beta_i^2)\sum_{\nu=1}^{n}(-1)^\nu \omega_i^{2\nu}\tilde{d}_\nu = \alpha_i^2 + \beta_i^2, \quad i = \overline{1,l}. \tag{10}$$

The only solution of Eqs. (10) (see [13]) is given by coefficients $\tilde{d}_\nu$ and $\tilde{k}_\mu$ of the polynomials

$$\begin{aligned}\tilde{d}(s^2) &= \sum_{\nu=1}^{n}\tilde{d}_\nu s^{2\nu} + 1 = d(s)d(-s) = d_n^2\prod_{\nu=1}^{n}\left(s - s_\nu^{[d]-}\right)\left(s - s_\nu^{[d]+}\right),\\ \tilde{k}(s^2) &= \sum_{\mu=0}^{m}\tilde{k}_\mu s^{2\mu} = k(s)k(-s) = k_m^2\prod_{\mu=1}^{m}\left(s - s_\mu^{[k]-}\right)\left(s - s_\mu^{[k]+}\right),\end{aligned} \tag{11}$$

where $s_\nu^{[d]-}$ and $s_\nu^{[d]+}$ ($\nu = \overline{1,n}$) are roots of the polynomial $\tilde{d}(s^2)$ with negative and positive real parts respectively. Similarly, we denote the roots of the polynomial $\tilde{k}(s^2)$ by $s_\mu^{[k]-}$ and $s_\mu^{[k]+}$ ($\mu = \overline{1,m}$).

Since roots of polynomials $d(s)$ and $k(s)$ have negative real parts by definition, their values will be given by $s_\nu^{[d]-}$ and $s_\mu^{[k]-}$ ($\nu = \overline{1,n},\ \mu = \overline{1,m}$) respectively, and then

$$d(s) = d_n\prod_{\nu=1}^{n}\left(s - s_\nu^{[d]-}\right) \qquad \text{and} \qquad k(s) = k_m\prod_{\mu=1}^{m}\left(s - s_\mu^{[k]-}\right). \tag{12}$$

Thus we can construct the following algorithm for finding the necessary object coefficients.

Algorithm 1.

1. Construct the following even degree polynomials:

$$\begin{aligned}\tilde{d}(s^2) &= \tilde{d}_n s^{2n} + \tilde{d}_{n-1}s^{2n-2} + \cdots + \tilde{d}_1 s^2 + 1,\\ \tilde{k}(s^2) &= \tilde{k}_m s^{2m} + \tilde{k}_{m-1}s^{2m-2} + \cdots + \tilde{k}_1 s^2 + \tilde{k}_0,\end{aligned}$$

   by solving Eqs. (10) with frequency parameters replaced with their estimates.
2. Find the roots $s_\nu^{[d]\pm}$ ($\nu = \overline{1,n}$) and $s_\mu^{[k]\pm}$ ($\mu = \overline{1,m}$) of polynomials $\tilde{d}(s^2)$ and $\tilde{k}(s^2)$ respectively and, taking only the ones that have negative real parts: $s_\nu^{[d]-}$ ($\nu = \overline{1,n}$) and $s_\mu^{[k]-}$ ($\mu = \overline{1,m}$), construct the desired polynomials (12).

4. DELAY IDENTIFICATION

We introduce the numbers

$$\phi_i = \mathrm{Re}\,w(j\omega_i) \quad \text{and} \quad \psi_i = \mathrm{Im}\,w(j\omega_i), \quad i = \overline{1,l}, \tag{13}$$

where $w(j\omega_i) = k(j\omega_i)/d(j\omega_i)$.

Expression (5) implies that

$$e^{j\omega_i\tau} = \frac{w(j\omega_i)}{w_\tau(j\omega_i)}. \tag{14}$$

Using definitions (6) and (13), we get

$$\cos \omega_i \tau = \frac{\alpha_i \phi_i + \beta_i \psi_i}{\alpha_i^2 + \beta_i^2} \quad \text{and} \quad \sin \omega_i \tau = \frac{\alpha_i \psi_i - \phi_i \beta_i}{\alpha_i^2 + \beta_i^2}, \quad i = \overline{1, l}, \tag{15}$$

which implies the following expression for finding the delay:

$$\tau(r) = \frac{1}{\omega_i} \left[\arctan \left(\frac{\alpha_i \psi_i - \phi_i \beta_i}{\alpha_i \phi_i + \beta_i \psi_i} \right) + \pi r \right], \quad i = \overline{1, l}, \quad r = 0, \pm 1, \pm 2, \ldots, \tag{16}$$

which gives an infinite number of solutions.

To find the unique delay value, we apply the following test signal to the object:

$$u_\lambda(t) = \rho_\lambda e^{\lambda t} \sin \omega_\lambda t, \tag{17}$$

where ρ_λ and ω_λ are given positive numbers and λ is a given number.

The object output $y_\lambda(t)$ excited by this input will be multiplied by $e^{-\lambda t}$ and serve as waster from the Fourier filter. Outputs of this filter

$$\hat{\alpha}_\lambda = \frac{2}{\rho_\lambda \bar{t}^\lambda} \int\limits_{t_F^\lambda}^{t_F^\lambda + \bar{t}^\lambda} e^{-\lambda t} y_\lambda(t) \cos \omega_\lambda t \, dt \quad \text{and} \quad \hat{\beta}_\lambda = \frac{2}{\rho_\lambda \bar{t}^\lambda} \int\limits_{t_F^\lambda}^{t_F^\lambda + \bar{t}^\lambda} e^{-\lambda t} y_\lambda(t) \sin \omega_\lambda t \, dt \tag{18}$$

(where t_F^λ is the time when filtering begins, and $\bar{t}^\lambda$ is its duration) are estimates of the object's transition function for $s = \lambda + j\omega_\lambda$ [13]:

$$w_\tau^\lambda(\lambda + j\omega_\lambda) = \frac{k(\lambda + j\omega_\lambda)}{d(\lambda + j\omega_\lambda)} e^{-(\lambda + j\omega_\lambda)\tau} = \alpha_\lambda + j\beta_\lambda. \tag{19}$$

Using identified polynomials (12), we compute

$$\phi_\lambda + j\psi_\lambda = \frac{k(\lambda + j\omega_\lambda)}{d(\lambda + j\omega_\lambda)}. \tag{20}$$

Using (20), we write Eq. (19) as

$$e^{-\lambda \tau} (\cos \omega_\lambda \tau - j \sin \omega_\lambda \tau) = \frac{\alpha_\lambda + j\beta_\lambda}{\phi_\lambda + j\psi_\lambda} \tag{21}$$

and decompose it into real and imaginary parts:

$$e^{-\lambda \tau} \cos \omega_\lambda \tau = \frac{\alpha_\lambda \phi_\lambda + \beta_\lambda \psi_\lambda}{\phi_\lambda^2 + \psi_\lambda^2} \qquad \text{and} \qquad e^{-\lambda \tau} \sin \omega_\lambda \tau = -\frac{\beta_\lambda \phi_\lambda - \alpha_\lambda \psi_\lambda}{\phi_\lambda^2 + \psi_\lambda^2}. \tag{22}$$

Adding up the squares of these equalities, we get the following expression:

$$e^{-2\lambda \tau} = \left(\frac{\alpha_\lambda \phi_\lambda + \beta_\lambda \psi_\lambda}{\phi_\lambda^2 + \psi_\lambda^2} \right)^2 + \left(\frac{\beta_\lambda \phi_\lambda - \alpha_\lambda \psi_\lambda}{\phi_\lambda^2 + \psi_\lambda^2} \right)^2, \tag{23}$$

which implies the following formula for computing the delay:

$$\tau = \frac{1}{2\lambda} \ln \left(\frac{\phi_\lambda^2 + \psi_\lambda^2}{\alpha_\lambda^2 + \beta_\lambda^2} \right). \tag{24}$$

5. IDENTIFYING CONVERGENCE CONDITIONS

To construct convergence conditions for our estimates of frequency parameters $\alpha_i(\overline{t})$ and $\beta_i(\overline{t})$ with respect to original values α_i and β_i $(i = \overline{1,l})$, we introduce the following function:

$$l_i^\alpha(\overline{t}) = \frac{2}{\rho_i \overline{t}} \int\limits_{t_F}^{\overline{t}+t_F} \overline{y}(t) \sin \omega_i t \, dt \quad \text{and} \quad l_i^\beta(\overline{t}) = \frac{2}{\rho_i \overline{t}} \int\limits_{t_F}^{\overline{t}+t_F} \overline{y}(t) \cos \omega_i t \, dt, \quad i = \overline{1,l}, \tag{25}$$

where $\overline{y}(t)$ is the "natural" control object output with no test signal at all (3) $(u(t) = 0)$. In this case, the object can only get excited with an external disturbance $f(t)$, and the measurement channel contains errors. $\eta(t)$.

Similar to [3], we introduce the following definitions.

Definition 5.1. An external disturbance $f(t)$ and a measurement noise $\eta(t)$ are called FF-filterable if for given numbers δ^α and δ^β there exists filtering time $\overline{t}^*$ such that

$$\frac{l_i^\alpha(\overline{t}^*)}{\alpha_i(\overline{t}^*)} \leqslant \delta^\alpha \quad \text{and} \quad \frac{l_i^\beta(\overline{t}^*)}{\beta_i(\overline{t}^*)} \leqslant \delta^\beta, \quad i = \overline{1,l}. \tag{26}$$

If filtering errors $\Delta\alpha_i = \alpha_i - \alpha_i(\overline{t})$, $\Delta\beta_i = \beta_i - \beta_i(\overline{t})$ $(i = \overline{1,l})$ satisfy $\lim_{\overline{t}\to\infty} \Delta\alpha_i = \lim_{\overline{t}\to\infty} \Delta\beta_i = 0$ $(i = \overline{1,l})$, we call external disturbance $f(t)$ and measurement noise $\eta(t)$ strictly FF-filterable.

To illustrate the notion of FF-filterability, consider the case when $f(t)$ and $\eta(t)$ can be decomposed into Fourier series, and $u(t) = 0$. Using (2), we rewrite (7) as

$$\alpha_i(\overline{t}) = \check{\alpha}_i(\overline{t}) + \sigma_i^\alpha(\overline{t}) \quad \text{and} \quad \beta_i(\overline{t}) = \check{\beta}_i(\overline{t}) + \sigma_i^\beta(\overline{t}), \quad i = \overline{1,l},$$

where

$$\begin{aligned} \check{\alpha}_i(\overline{t}) &= \frac{2}{\rho_i \overline{t}} \int\limits_{t_F}^{t_F+\overline{t}} y(t) \sin \omega_i t \, dt, \quad \check{\beta}_i(\overline{t}) = \frac{2}{\rho_i \overline{t}} \int\limits_{t_F}^{t_F+\overline{t}} y(t) \cos \omega_i t \, dt, \\ \sigma_i^\alpha(\overline{t}) &= \frac{2}{\rho_i \overline{t}} \int\limits_{t_F}^{t_F+\overline{t}} \eta(t) \sin \omega_i t \, dt, \quad \sigma_i^\beta(\overline{t}) = \frac{2}{\rho_i \overline{t}} \int\limits_{t_F}^{t_F+\overline{t}} \eta(t) \cos \omega_i t \, dt, \end{aligned} \qquad i = \overline{1,l}.$$

Functions $\sigma_i^\alpha(\overline{t})$ and $\sigma_i^\beta(\overline{t})$ are caused by measurement noise $\eta(t)$.

Functions $f(t)$ and $\eta(t)$ can be decomposed into a Fourier series and can be represented as

$$f(t) = \sum_{r=1}^{n_1} a_r \sin \omega_r^f t \qquad \text{and} \qquad \eta(t) = \sum_{p=1}^{n_2} b_p \sin \omega_p^\eta t,$$

where n_1, a_r, ω_r^f $(r = \overline{1,n_2})$ and n_2, b_p, ω_p^η $(p = \overline{1,n_1})$ are unknown numbers. If they do not contain frequencies that coincide with the frequencies of the test signal: $\omega_r^f \neq \omega_i$ and $\omega_p^\eta \neq \omega_i$ $(r = \overline{1,n_1},\ p = \overline{1,n_2},\ i = \overline{1,l})$, then these functions are strictly FF-filterable. This fact can also be written as

$$\lim_{\overline{t}\to\infty} \sigma_i^\alpha(\overline{t}) = 0, \quad \lim_{\overline{t}\to\infty} \sigma_i^\beta(\overline{t}) = 0, \quad \lim_{\overline{t}\to\infty} \check{\alpha}_i(\overline{t}) = \alpha_i, \quad \lim_{\overline{t}\to\infty} \check{\beta}_i(\overline{t}) = \beta_i, \quad i = \overline{1,l}.$$

If test signal frequencies coincide with frequencies of the functions $f(t)$ and $\eta(t)$, we get that

$$\begin{aligned} \lim_{\overline{t}\to\infty} \sigma_i^\alpha(\overline{t}) &= \epsilon_i^\alpha, \quad \lim_{\overline{t}\to\infty} \sigma_i^\beta(\overline{t}) = \epsilon_i^\beta, \\ \lim_{\overline{t}\to\infty} \check{\alpha}_i(\overline{t}) &= \alpha_i + \xi_i^\alpha, \quad \lim_{\overline{t}\to\infty} \check{\beta}_i(\overline{t}) = \beta_i + \xi_i^\beta, \end{aligned} \qquad i = \overline{1,l},$$

where ϵ_i^α, ϵ_i^β and ξ_i^α, ξ_i^β $(i = \overline{1,l})$ are some positive numbers.

If the numbers ϵ_i^α, ϵ_i^β and ξ_i^α, ξ_i^β $(i = \overline{1,l})$ are sufficiently small, we call functions $f(t)$ and $\eta(t)$ FF-filterable.

6. JOINT IDENTIFICATION OF COEFFICIENTS AND DELAY

Since we need different test signals for the identification of object coefficients and delay, we divide identification into two stages. On the first stage, we supply test signal (3) to the object and identify only object coefficients; on the second stage, we supply (17) and identify the value of delay.

Joint identification of object coefficients and delay is also possible. To do so, we need to use a test signal of the form

$$u(t) = \rho_\lambda e^{\lambda t} \sin \omega_\lambda t + \sum_{i=1}^{l} \rho_i \sin \omega_i t, \qquad \omega_\lambda \neq \omega_i, \quad i = \overline{1,l}, \tag{27}$$

on the object.

To find estimates for frequency parameters, we need to use two filters, (7) and (18), simultaneously.

If we choose the number λ with a negative value, the signal with exponential component will not have a strong negative effect on the identification of object coefficients.

7. EXAMPLE

This example is based on the two-stage identification algorithm.

Consider an object defined by equation

$$0.7\ddot{y}(t) + 0.8\dot{y}(t) + y(t) = 0.4\dot{u}(t-3) + u(t-3) + f(t), \tag{28}$$

where the external disturbance is

$$f(t) = 2\mathrm{sgn}[\sin(5t)].$$

Measurement noise $\eta(t)$ produces random values uniformly distributed on the interval $[-0.05; 0.05]$.

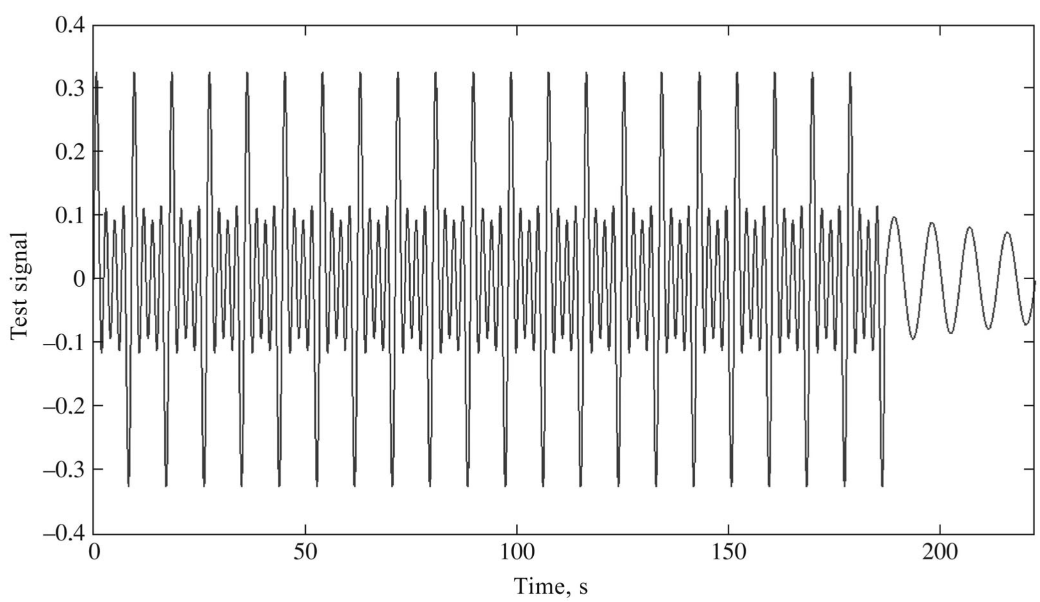

Fig. 1. Test signal.

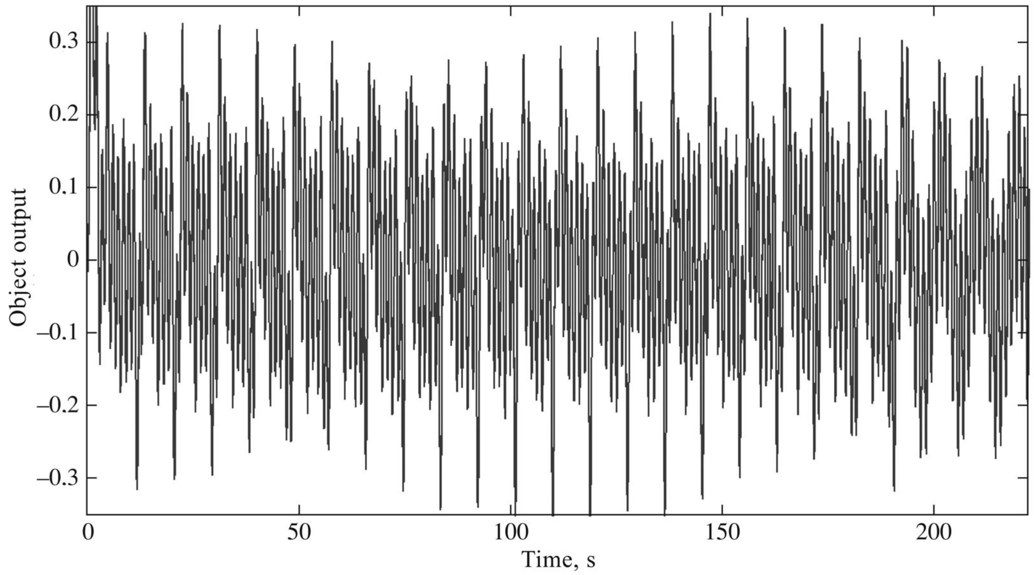

Fig. 2. Object output under the influence of the test signal.

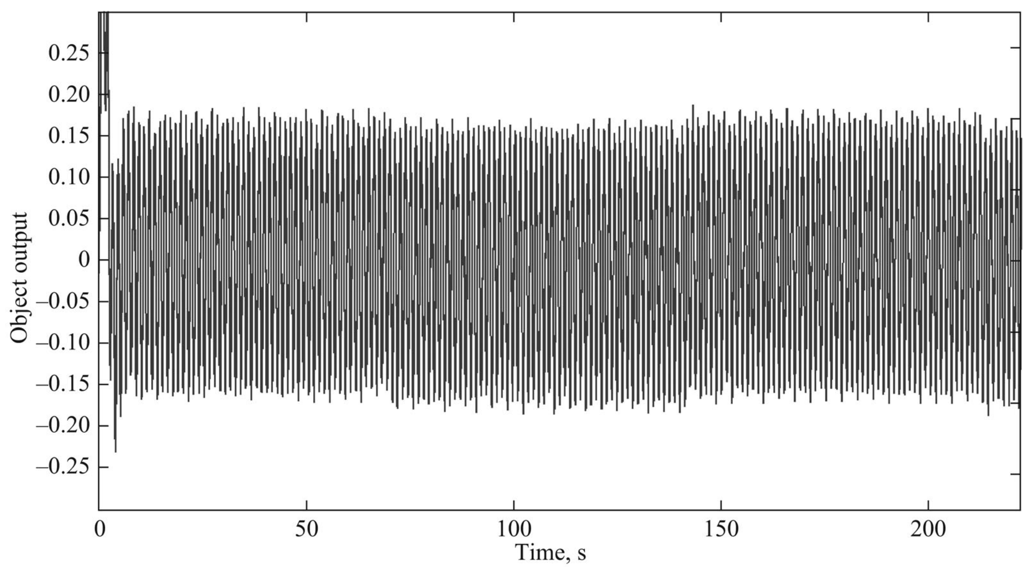

Fig. 3. Object output without the influence of the test signal.

Such a noise is possible since the numerical solution of Eqs. (28) was done with its discrete model with discretization interval $h = 0.01$.

Test signal:

$$u(t) = 0.05\sin(0.707t) + 0.075\sin(1.41t) + 0.1\sin(2.12t) + 0.15\sin(2.83t). \quad (29)$$

For the identification delay, we choose a test signal with frequency $\omega_\lambda = 0.707$:

$$u_\tau(t) = 0.1e^{-0.01t}\sin(0.707t). \quad (30)$$

After identification we get the following object:

$$0.65\ddot{y}(t) + 0.83\dot{y}(t) + y(t) = 0.32\dot{u}(t - 3.48) + 1.06u(t - 3.48) + f(t). \quad (31)$$

Graphs of the test signal and object output are shown on Figs. 1 and 2 respectively. For comparison, Fig. 3 shows the graph of object output without the influence of the test signal, and changes in the output occur under the influence of external disturbances. Figures 2 and 3 show that the test signal introduces negligible changes in the output of the object being identified.

8. CONCLUSION

In this work, we have extended the finite-frequency identification approach to objects with delay. We have obtained identification equations that let us find the control object coefficients independently of the delay. We have proposed a method for uniquely determining the delay.

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