FREQUENCIAL ADAPTIVE PDD-CONTROLLER

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Keywords : adaptive control, unknown-but-bounded disturbance, PDD-controller.

Abstract

Algorithm of adaptive PDD-controller for plant with unknown coefficients and pulse-width modulation (PWM) in the presence of unknown-but-bounded disturbance is derived. It makes use of a sufficient small test signal. Adaptation process consists of intervals on which a plant and closed-loop system are identified on the base of finite-frequency identification technique. The plant coefficients estimates are used for controller redesign and the identified characteristic polynomial of closed-loop system is compared with a specified wished characteristic polynomial. If these polynomials are close then adaptation is ended. The ways of design of admissible test signal are proposed.

1 INTRODUCTION

Selftuning and adaptive PI- and PID-controllers [1] are widely used for technological process control [1, 2]. Their algorithms are based on different variants of the least squares techniques in which the measurement noise and external disturbance are assumed to be "white noise" [3]. In practice such assumption does not take place often and so these controllers may lose serviceability. In last decade adaptive control methods in which disturbances are unknown-but-bounded functions have been developed. These techniques are based on recurrent aimed inequalities [4], least squares estimation algorithm with dead zone [5], frequency domain parameters that are found by Fourier's filters making use of output of a plant excited by test signal [6, 7].

In paper [8] algorithm for adaptive PID-controller design of linear system using the third approach was proposed.

However in practice there is often making use of pulse-width modulation of signal applied to electrodrive. In this case PID-controller is replaced with a PDD-controller.

In the present paper the adaptive control method based on use of frequency domain parameters is developed on a case of systems with the pulse-width modulation.

As an example the application of an adaptive PDD-controller for a real impulse system of temperature stabilisation of air [9] with unknown parameters of plant and unknown-but-bounded disturbances is described.

2 Problem statement

Consider a completely controllable and asymptotically stable plant described by the following differential equation

$$\ddot{y} + 2\xi\omega_0\dot{y} + \omega_0^2 y = \omega_0^2 k_{pl} T_1 \dot{\mu} + \omega_0^2 k_{pl}\mu + f, \qquad (1)$$

where $y(t)$ is measured plant output, $f(t)$ is an unknown-but-bounded disturbance, undamped natural frequency ω_0, damping ratio ξ, coefficient k_{pl} and time constant T_1 are unknown numbers, μ is output of the executive device described by the following equations of electrodrive with pulse-width modulation

$$\dot{\mu} = k_{dr}\nu, \qquad (2)$$

$$\nu(t) = \begin{cases} a\,\mathrm{sign}\,u(t) & \text{for } (l-1)H \le t < (l-1)H + t_{ul} \\ 0 & \text{for } (l-1)H + t_{ul} \le t < lH, \end{cases} \qquad (3)$$

where

$$t_{ul} = \frac{H}{u_{\max}} \left| u[(l-1)H] \right|, \quad l = 1, 2, \ldots,$$

k_{dr} is a coefficient, amplitude a and interval H are given numbers, $u(t)$ is an output of controller, $u_{\max} = \max|u(t)|$ is a known number, $u[(l-1)H]$ is a value of $u(t)$ in the time moment $t = (l-1)H$. Further it is assumed that interval H is sufficiently small.

The diagram of dependence (3) is given in fig. 1.

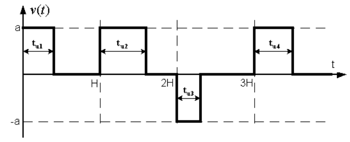

Fig. 1

The controller is described by following equation

$$T_{2p}\ddot{u} + T_{1p}\dot{u} + u = k_p\left[T_g\ddot{e} + \dot{e} + \frac{1}{T_u}e\right], \qquad (4)$$

it is named PDD-controller [10], [11]. In equation (4) $e = y - y_r$ is a tracking error, $y_r(t)$ is a reference input, T_{1p}, T_{2p} are sufficiently small numbers. The controller coefficients k_p, T_u, T_g, T_{1p} and T_{2p} are found as adaptation result.

Equation (2) , (3) and (4) are described the impulse analog of the PID-controller.

For convenience equations (1) and (2) may be written as:

$$\dddot{y} + d_2\ddot{y} + d_1\dot{y} = k_1\dot{\nu} + k_0\nu + \dot{f}, \qquad (5)$$

where

$$d_2 = 2\xi\omega_0,\ d_1 = \omega_0^2,\ k_1 = \omega_0^2 k_{pl}k_{dr}T_1,\ k_0 = \omega_0^2 k_{pl}k_{dr} \qquad (6)$$

and controller (4) is described by the equation with piecewise-constant coefficients

$$g_2^{[i]}\ddot{u} + g_1^{[i]}\dot{u} + g_0^{[i]}u = r_2^{[i]}\ddot{e} + r_1^{[i]}\dot{e} + r_0^{[i]}e + v^{[i]}, \quad t_{i-1} \le t \le t_i, \quad i = \overline{1,N} \qquad (7)$$

$$g_0^{[i]} = T_u^{[i]},\ g_1^{[i]} = T_u^{[i]}T_{1p}^{[i]},\ g_2^{[i]} = T_u^{[i]}T_{2p}^{[i]}, \quad r_0^{[i]} = k_p^{[i]},\ r_1^{[i]} = k_p^{[i]}T_u^{[i]},\ r_2^{[i]} = k_p^{[i]}T_u^{[i]}T_g^{[i]}. \qquad (8)$$

where i $\left(i = \overline{1,N}\right)$ is an adaptation interval number; the time of ending of each intervals t_i, their amount N and numbers $g_k^{[i]}$, $r_k^{[i]}$ $\left(k = \overline{0,2}\right)$ are found in adaptation process.

Duration of adaptation intervals $\delta_i = t_i - t_{i-1}$ $\left(i = \overline{1,N}\right)$ have to satisfy the conditions

$$\delta_i \ge \delta_{i-1} + \delta^*, \quad i = \overline{1,N} \qquad (9)$$

where δ^* is a given positive number.

In equation (7)

$$v^{[i]}(t) = \sum_{k=1}^{4}\rho_k \sin\omega_k(t - t_{i-1}),\ t_{i-1} \le t \le t_i \quad i = \overline{1,N} \qquad (10)$$

are the test signals with test frequencies $\omega_k^{[i]} > 0$ and test amplitudes $\rho_k^{[i]} > 0$, $\left(k = \overline{1,4},\ \ i = \overline{1,N}\right)$.

After adaptation ending the controller is described by equation (7) in that $g_i = g_i^{[N]}$, $r_i = r_i^{[N]}$ $\left(i = \overline{0,2}\right)$ and then this equation has following form

$$g_2\ddot{u} + g_1\dot{u} + g_0 u = r_2\ddot{e} + r_1\dot{e} + r_0 e, \qquad (11)$$

For simplicity, further one is considered two species of the intervals: *the first species* when the test signals are directly applied to the PWM block (3):

$$u = v^{[i]}(t) = \sum_{k=1}^{3}\rho_k \sin\omega_k(t - t_{i-1}) \quad t_{i-1} \le t < t_i \quad i \in \overline{1,N} \qquad (12)$$

(in this case: $g_k^{[i]} = 0$, $k = 1,2$, $g_0^{[i]} = 1$, $r_k^{[i]} = 0$ $\left(k = \overline{0,2}\right)$) and intervals of *second species in* which

$$v^{[i]}(t) = \sum_{k=1}^{4}\tilde{\rho}_k \sin\tilde{\omega}_k(t - t_{i-1}), \quad t_{i-1} \le t < t_i \quad i \in \overline{2,N}. \qquad (13)$$

The test frequencies are multiply to minimal frequencies ω_1 and $\tilde{\omega}_1$: $\omega_2 = n_1\omega_1$, $\omega_3 = n_2\omega_1$, $\tilde{\omega}_2 = \tilde{n}_1\tilde{\omega}_1$, $\tilde{\omega}_3 = \tilde{n}_2\tilde{\omega}_1$, $\tilde{\omega}_4 = \tilde{n}_3\tilde{\omega}_1$, where n_1, n_2, $\tilde{n}_1$, $\tilde{n}_2$, $\tilde{n}_3$ are integers.

The test signals amplitudes are found in adaptation process from the following conditions of "small excitation" of plant and controller outputs

$$\left|y^{[i]}(t) - \bar{y}^{[i]}(t)\right| \le \varepsilon_y,\ \left|u^{[i]}(t) - \bar{u}^{[i]}(t)\right| \le \varepsilon_u \quad i = \overline{1,N} \qquad (14)$$

where $\bar{y}^{[i]}(t)$ $\bar{u}^{[i]}(t)$ are outputs of plant (5) and controller (7) when the test signals are absent ($v^{[i]} = 0$), ε_y ε_u are sufficiently small numbers which express a requirement of smallness of component excited by the test signal in comparison with components because of reference signal and disturbance. The test signals (12) and (13) are called *admissible* if they satisfy the conditions (14).

The frequencies and amplitudes of signals (12) and (13) are assumed to be known. The way of their determination by experiment and convergence of adaptation process is described in [8].

When interval H is sufficiently small characteristic polynomial of system (5),(3),(11) is

$$\begin{aligned}\varphi(s) &= (s^3 + d_2 s^2 + d_1 s)s(g_2 s^2 + g_1 s + g_0) - \\ &- (k_1 s + k_0)(r_2 s^2 + r_1 s + r_0) = \\ &= \varphi_5 s^5 + \varphi_4 s^4 + \varphi_3 s^3 + \varphi_2 s^2 + \varphi_1 s + \varphi_0.\end{aligned} \qquad (15)$$

Let the requirements to the settling time, maximum overshoot and steady-state error of system (5),(3),(11) be assigned by the coefficients of a wished characteristic polynomial

$$\psi(s) = \psi_5 s^5 + \psi_4 s^4 + \psi_3 s^3 + \psi_2 s^2 + \psi_1 s + \psi_0, \qquad (16)$$

Problem 2.1 Find an adaptation algorithm for coefficients of controller (7) such that the characteristic polynomial (15) of system (5),(3),(11) and a specified Hurwitz's polynomial (16) meet the following demands

$$|\psi_i - \varphi_i| \le \varepsilon_i^{\phi} \quad i = \overline{0,5}, \qquad (17)$$

where ε_i^{ψ}, $(i = \overline{0,5})$ are given numbers.

Remarks

- The coefficients of plant (1) is assumed to be constant. However, algorithm solved problem 2.1 is small changed if plant coefficients and reference input value $\bar{y}_r$ are changed in some time moment $t^{(1)} > t_N$. In this case controller (7) have to contain a test signal in order to determine $t^{(1)}$.
- In practice, measurement results of plant output is $\tilde{y}(t) = y(t) + \eta(t)$ where $\eta(t)$ is unknown-but-bounded measurement noise. In problem 2.1 the noise is not took into account for simplicity and a proposed algorithm of this problem solution does not change in the case of noise.
- Equations (1) allows indirectly to take account of plant with delay In fact, if $\xi > 1$ then plant transfer function

$$w(s)=\frac{k(s)}{d(s)}=\frac{\omega_0^2 k_p(T_1 s+1)}{s^2+2\xi\omega_0+\omega_0^2}=\frac{k_p(T_1 s+1)}{(T_2 s+1)(T_3 s+1)} \quad (18)$$

under $T_1 = -T_3$ contains approximation of transfer function $e^{-0.5T_3 s}$.

3 The first adaptation interval (controller initialisation)

3.1 Plant output filtering

On the first interval test signals are applied to the PWM block (3). The outputs of "plant" (5) are applied to Fourier's filter,

$$\begin{aligned} \hat{\alpha}_k = \alpha_k(\delta) = \frac{2}{\rho_k \tau}\int_{t_F}^{t_F+\tau} y(t)\sin\omega_k(t-t_0)\,dt \\ \hat{\beta}_k = \beta_k(\delta) = \frac{2}{\rho_k \tau}\int_{t_F}^{t_F+\tau} y(t)\cos\omega_k(t-t_0)\,dt \end{aligned} \quad k=\overline{1,3} \quad (19)$$

whose outputs are measure in moment time $\delta = qT$, where $T = \frac{2\pi}{\omega_1}$ is a basic period, $q = 1, 2, \ldots$; t_F is a filtering start time (for simplicity, $t_F = \tilde{q}T$, $\bar{q}$ is a given number), $\delta = t_F + \tau$, τ is a filtering time.

For fixed value δ numbers $\hat{\alpha}_k$ and $\hat{\beta}_k$ $(k = \overline{1,3})$ are the estimates of frequency domain parameters (FDP) α_k and β_k $(k = \overline{1,3})$ that are connected with plant transfer function by expressions:

$$\alpha_k = \operatorname{Re} w(j\omega_k), \quad \beta_k = \operatorname{Im} w(j\omega_k), \quad k = \overline{1,3} \quad (20)$$

3.2 Frequency equations (plant identification)

The FDP estimates $\hat{\alpha}_k = \alpha_k(qT)$, $\hat{\beta}_k = \beta_k(qT)$ $(k = \overline{1,3})$ are coefficients of the following frequency equations of identification

$$\begin{aligned} &[(j\omega_k)\hat{k}_1+\hat{k}_0]-(\hat{\alpha}_k+j\hat{\beta}_k)[(j\omega_k)^2\hat{d}_2+(j\omega_k)\hat{d}_1)]= \\ &=(\hat{\alpha}_k+j\hat{\beta}_k)(j\omega_k)^3 \quad k=\overline{1,2}, \end{aligned} \quad (21)$$

where $\hat{d}_i$, $\hat{k}_j$ $(i = 1, 2,\ j = 0, 1)$ are coefficients estimates of "plant" (5).

These equations follow from the identity

$$k(s) - w(s)d(s) = 0$$

(under $s = j\omega_k$ $(k = \overline{1,3})$) and expressions (20) after substitution of FDP by their estimations (19).

The frequency equations (21) may be written as

$$\begin{aligned} \hat{k}_0 + \omega_k\hat{\alpha}_k\hat{d}_2 + \omega_k\hat{\beta}_k\hat{d}_1 = \hat{\beta}_k\omega_k^3 \\ \omega_k\hat{k}_1 - \hat{\alpha}_k\hat{d}_1 + \omega_k\hat{\beta}_k d_2 = -\alpha_k\omega_k^3 \end{aligned} \quad k=\overline{1,3}. \quad (22)$$

3.3 Determination of interval ending moment

Solving system (22) for the different values of FDP $\alpha_k(qT)$, $\beta_k(qT)$ ($k = \overline{1,3}$, $q = \bar{q}+1, \bar{q}+2\ldots$) the plant coefficients estimates $d_i(qT)$, $k_j(qT)$ $(i=1, 2,\ j=0, 1,$ $q = \bar{q}+1, \bar{q}+2\ldots)$ are found.

The first interval duration is determined by the following necessary conditions of identification convergence

$$\begin{aligned} |\omega_0(qT) - \omega_0[(q-1)T]| &\le \varepsilon_{11}, \\ |\xi(qT) - \xi[(q-1)T]| &\le \varepsilon_{21}, \\ |T_1(qT) - T_1[(q-1)T]| &\le \varepsilon_{31}, \\ |\kappa_p(qT) - \kappa_p[(q-1)T]| &\le \varepsilon_{41}, \end{aligned} \quad q=\bar{q}+1, \bar{q}|+2\ldots \quad (23)$$

where ε_{i1} $(i = \overline{1,4})$ are given sufficiently small numbers.

Let for $q = q_1$ these conditions hold then ending time of the first interval is $t_1 = q_1 T$.

3.4 Controller coefficients calculation

Let's consider the Bezout-Identity

$$\begin{aligned} &(s^2+\hat{d}_2 s+\hat{d}_1)s(g_2 s^2+g_1 s+g_0)-(\hat{k}_1 s+\hat{k}_0)\times \\ &\times(r_2 s^2+r_1 s+r_0)=s^5+\psi_4 s^4+\psi_3 s^3+\psi_2 s^2+\psi_1 s+\psi_0, \end{aligned} \quad (24)$$

that gives the system of the linear algebraic equations

$$\begin{aligned} \hat{g}_2 = 1, \quad \hat{g}_1 + \hat{d}_2 g_2 = \psi_4, \\ \hat{g}_0 + \hat{d}_2 g_1 + \hat{d}_1 g_2 - \hat{k}_1 r_2 = \psi_3, \\ \hat{d}_2 g_0 + \hat{d}_1 g_1 - \hat{k}_1 r_1 - \hat{k}_0 r_2 = \psi_2, \\ \hat{d}_1 g_0 - \hat{k}_1 r_0 - \hat{k}_0 r_1 = \psi_1, \quad -\hat{k}_0 r_0 = \psi_0. \end{aligned} \quad (25)$$

Solving these equations the coefficients of controller

$$(g_2^{[2]}s^2 + g_1^{[2]}s + g_0^{[2]})u = (r_2^{[2]}s^2 + r_1^{[2]}s + r_0^{[2]})e + v^{[2]} \quad (26)$$

are found.

4 The second adaptation interval

4.1 Closed-loop system

On the second adaptation interval "plant" (5) is closed by the controller (26). The closed-loop system (5) , (26) may be written as

$$\varphi^{[2]}(s)y = (k_1 s + k_0)v^{[2]} + \tilde{f}, \quad (27)$$

where

$$\begin{aligned}\varphi^{[2]}(s) &= (s^2+d_2s+d_1)s(g_2^{[2]}s^2+g_1^{[2]}s+g_0^{[2]})- \\ &-(k_1s+k_0)(r_2^{[2]}s^2+r_1^{[2]}s+r_0^{[2]}) = \\ &= \varphi_5^{[2]}s^5+\varphi_4^{[2]}s^4+\varphi_3^{[2]}s^3+\varphi_2^{[2]}s^2+\varphi_1^{[2]}s+\varphi_0^{[2]}\end{aligned} \quad (28)$$

$$\begin{aligned}\tilde{f}(t) &= (g_2^{[2]}s^2+g_1^{[2]}s+g_0^{[2]})\dot{f}-(k_1s+k_0)\times \\ &\times(r_2^{[2]}s^2+r_1^{[2]}s+r_0^{[2]})y_r\end{aligned}$$

,

Equation (27) may be interpreted as "new plant" with the following transfer function

$$w_{cl}(s) = k(s)/\varphi^{[2]}(s) \quad (29)$$

Frequency domain parameters corresponding to this transfer function are called FDP of a closed-loop system and they described by expressions:

$$\theta_k = \mathrm{Re}\, w_{cl}(j\tilde{\omega}_k), \gamma_k = \mathrm{Im}\, w_{cl}(j\tilde{\omega}_k), \quad (30)$$

4.2 Closed-loop output filtering

Estimates of closed-loop system FDP are found as outputs of Fourier's filter

$$\left.\begin{aligned}\hat{\theta}_k &= \theta_k(\delta) = \frac{2}{\tilde{\rho}_k\tau}\int_{t_F}^{t_F+\tau} y(t)\sin\tilde{\omega}_k\, dt \\ \hat{\gamma}_k &= \gamma_k(\delta) = \frac{2}{\tilde{\rho}_k\tau}\int_{t_F}^{t_F+\tau} y(t)\cos\tilde{\omega}_k\, dt\end{aligned}\right. \quad k=\overline{1,4}, \quad (31)$$

where $y(t)$ is output of system (5),(3),(26) excited by test signal (13), $t_F = \bar{\bar{q}}\tilde{T} = \bar{\bar{q}}\frac{2\pi}{\tilde{\omega}_1}$, $\bar{\bar{q}}$ is a given number.

4.3 Frequency equation solution (the closed-loop system identification)

The frequency equations for identification of "plant" (27) are

$$\begin{aligned}&[j\tilde{\omega}_k\hat{k}_1 + \hat{k}_0] - (\hat{\theta}_k + j\hat{\gamma}_k)\times\Big[(j\tilde{\omega}_k)^4\hat{\varphi}_4^{[2]}+ \\ &+(j\tilde{\omega}_k)^3\hat{\varphi}_3^{[2]}+(j\tilde{\omega}_k)^2\hat{\varphi}_2^{[2]}+(j\tilde{\omega}_k)\hat{\varphi}_1^{[2]}+\hat{\varphi}_0^{[2]}\Big] = \\ &= (\hat{\nu}_k + j\hat{\mu}_k)(j\tilde{\omega}_k)^5, \quad \varphi_5^{[2]} = 1\end{aligned} \quad (32)$$

The solution of this system for the different values of FDP: $\hat{\theta}_k = \theta_k(qT)$, $\hat{\gamma}_k = \gamma_k(qT)$ gives estimates $\varphi_i^{[2]}(qT)$ of characteristic polynomial coefficients $\varphi_i^{[2]}(q\tilde{T})$ $(i=\overline{0,5},\ q=\bar{\bar{q}}+1, \bar{\bar{q}}+2,\ldots)$.

4.4 Target condition examination

If there exists number $q = q_2$ such that the target conditions

$$\left|\psi_i - \hat{\varphi}_i^{[2]}\right| \leq \varepsilon_i^{\psi} \quad i = \overline{0,5} \quad (33)$$

hold then $N = 2$, the second adaptation interval is ended and controller (7) has coefficients $g_i = g_i^{[2]}$, $r_i = r_i^{[2]}$ $(i = \overline{0,2})$.

4.5 Controller coefficients calculation

If number $q = q_2$ for that the inequalities (33) hold does not exist (it means that identification accuracy obtained on the first interval is not sufficiently) then FDP are used for an improvement of plant FDP estimates.

These two species of FDP are linked by the following expressions

$$\alpha_k + j\beta_k = \frac{\theta_k + j\gamma_k}{(\theta_k + j\gamma_k)w_c(j\tilde{\omega}_k) + w_e(j\tilde{\omega}_k)} \quad k = \overline{1,3} \quad (34)$$

where

$$w_c(s) = \frac{r_2^{[2]}s^2+r_1^{[2]}s+r_0^{[2]}}{(g_2^{[2]}s^2+g_1^{[2]}s+g_0^{[2]})s}, \quad w_e(s) = \frac{1}{(g_2^{[2]}s^2+g_1^{[2]}s+g_0^{[2]})s}. \quad (35)$$

Substituting in (34) θ_k and γ_k $(k = \overline{1,2})$ by their estimates $\theta_k(q\tilde{T})$, $\gamma_k(q\tilde{T})$ ($k = \overline{1,3}$, $q = \bar{\bar{q}} + \tilde{q}_2, \bar{\bar{q}} + \tilde{q}_2+1, \ldots$) FDP estimates of "plant" (5) $\hat{\alpha}_k = \alpha_k(q\tilde{T})$, $\hat{\beta}_k = \beta_k(q\tilde{T})$ ($k = \overline{1,3}$) are calculated. Here, in according to (9) $\tilde{q}_2$ is found from the following inequality $(\bar{\bar{q}}+\tilde{q}_2)\tilde{T} \geq q_1T + \delta^*$.

Solving frequency equation (22) under $\omega_k = \tilde{\omega}_k$ $(k = \overline{1,3})$ new estimates $d_i(q\tilde{T})$, $k_j(q\tilde{T})$ $(i = 1, 2,$ $j = 0, 1$, $q = \bar{\bar{q}}+\tilde{q}_2, \bar{\bar{q}}+\tilde{q}_2+1, \ldots)$ are found.

Then necessary conditions

$$\left|\begin{aligned}&\left|\omega_0(q\tilde{T})-\omega_0[(q-1)\tilde{T}]\right| \leq \varepsilon_{12}, \quad \left|\xi(q\tilde{T})-\xi[(q-1)\tilde{T}]\right| \leq \varepsilon_{22} \\ &\left|T_i(q\tilde{T})-T_i[(q-1)\tilde{T}]\right| \leq \varepsilon_{32}, \quad \left|\kappa_p(q\tilde{T})-\kappa[(q-1)\tilde{T}]\right| \leq \varepsilon_{42}\end{aligned}\right. \quad (36)$$

are examined. Here ε_{i2}, $(i=\overline{1,4})$ are given numbers.

If for $q=q_2$ these conditions hold then ending time of the second interval $t_2=q_2\tilde{T}$.

Making use of coefficients of "plant"(5) $\hat{d}_i = d_i(q_2\tilde{T})$ $\hat{k}_i = k_i(q_2\tilde{T})$ $(i=\overline{1,2})$ the equations (25) are solved and coefficients of controller

$$(g_2^{[3]}s^2+g_1^{[3]}s+g_0^{[3]})u=(r_2^{[3]}s^2+r_1^{[3]}s+r_0^{[3]}s)y+v^{[3]} \quad (37)$$

are found and so one.

5 Adaptation algorithm

Algorithm 5.1 (algorithm of frequencial adaptive PDD-controller) contains the following steps: a) find amplitudes of test signal (12) making use of algorithm 5.1 [8], apply output of plant (5) excited by this test signal to Fourier filter (19) and measure its outputs in the time moment $\delta=qT$, $q=\bar{q}+1, \bar{q}+2, \ldots$;

b) solve frequency equations (22) for these time moments, examine necessary conditions (23) and find time $t_1=q_1T$;

c) calculate coefficients of controller (26) solving equations (25);

d) close plant by controller (26), find amplitudes of test signal (13) on the base of algorithm like algorithm 5.1 of [8], excite system (5),(3),(26) by test signal (13) and measure Fourier's filter outputs (31) in time moments $\delta=q\tilde{T}$ $(q=\bar{\bar{q}}+\tilde{q}_2, \bar{\bar{q}}+\tilde{q}_2+1, \ldots)$.

e) solve frequency equations (32) for this time moments and examine target conditions (33);

f) if these conditions do not satisfy then calculate plant FDP estimations by formulae (34), go to step (b) and so on.

6 Example

Consider a plant described by the following equations

$$\ddot{y}+d_2\dot{y}+d_1y=k_1\dot{\mu}+k_0\mu+f, \quad (38)$$

$$\dot{\mu}=\nu \quad (39)$$

and by the equation (3)

The values of parameters of the PWM block are

$$a=220,\ H=10\ \text{c},\ u_{\max}=220. \quad (40)$$

Remark. *The coefficients and disturbance of a real plant used under the simulation have the following values*

$$d_2=0.021,\ d_1=2\cdot10^{-5},\ k_1=-5\cdot10^{-6},\ k_0=10^{-7}, \quad (41)$$
$$f=-120\cos 0,00835t$$

Its transfer function is

$$w(s)=\frac{5\cdot10^{-3}(-50s+1)}{(10^3s+1)(50s+1)}, \quad (42)$$

■

Problem is to find coefficients of PDD-controller (7) such that characteristic polynomial of system (38),(40),(11) and the following wished polynomial

$$\psi(s)=(100s+1)(150s+1)(200s+1)(250s+1)(300s+1)= =0.225\cdot10^{12}s^5+0.6525\cdot10^1s^4+0.725\cdot10^8s^3+ +0.3875\cdot10^6s^2+0.1\cdot10^4s^1+0.1\cdot10^1 \quad (43)$$

meet the demands

$$|\psi_i-\varphi_i|\le 0,5\psi_i \qquad i=\overline{0,5} \quad (44)$$

To realise the numerical experiments on simulation of the adaptive process with a PWM the program "Frequency adaptive control with a PWM" specially was developed [12]. It operates in structure of a package ADAPLAB [13].

Below results of numerical experiments are given.

Step ( )

$$u=v^{[1]}=3\sin 0.002t+15\sin 0.004t+220\sin 0.06t, \quad (45)$$

$\bar{q}=4,\ T=3141,56$ sec.

Step (b) $q_1=28$.

Step (c)

$$g_2^{[2]}=0.225\cdot10^{12},\quad g_1^{[2]}=0.14346\cdot10^1, \quad g_0^{[2]}=0.25134\cdot10^8,\quad r_2^{[2]}=0.16920\cdot10^{13}, \quad r_1^{[2]}=-0.49280\cdot10^1,\quad r_0^{[2]}=-0.93197\cdot10^7. \quad (46)$$

Step (d)

$$v^{[2]}=2.2\cdot10^8(\sin 0.002t+\sin 0.006t+\sin 0.009t+ +\sin 0.012t), \quad \bar{\bar{q}}=2,\quad \tilde{T}=3141,56sec. \quad (47)$$

Step (e) Under $q_2=98$

$$\varphi^{[2]}(s)=0.225*10^{12}*s^5+0.87364*10^{10}*s^4+ +0.90511*10^8*s^3+0.49895*10^6*s^2+ +0.12644*10^4*s^1+1.2841$$

The target condition

$$\left|\psi_i-\hat{\varphi}_i^{[2]}\right|\le 0,5\psi_i \quad i=\overline{0,5}. \quad (48)$$

is hold and therefore PDD-controller (7) with coefficients (46) is resolved the adaptive control problem.

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