FINITE-FREQUENCY METHOD OF IDENTIFICATION

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Abstract. The identification method for the stable and unstable linear continuous plants in the presence of bounded disturbance is proposed. It bases self upon a notion of the frequency domain parameters that are signals of Fourier's filter outputs.
It is shown convergence of identification process and derived estimations of filtering errors.
Ways of identification time reduce are investigated.

Key Words. Identification, linear plant, frequency response, disturbance rejection, convergence.

1. INTRODUCTION

The frequency method of identification and adaptation in which a plant is excited by the harmonic test signals and its output applies to Fourier's filter has the deep roots (Eykhoff, 1974; Krasovskii, 1963).

The application area of the frequency approach is restricted by the stable plants under condition that an external disturbance does not contain the frequencies of the test signals.

If experiment scheme is changed by way of multiplying of a test signal by $e^{\lambda t}$ and the plant output by $e^{-\lambda t}$ ($\lambda>0$) then Fourier's filter outputs signals converge to some values named the frequency domain parameters (FDP).

FDP are a description of the stable and unstable plants and they may be used for the adaptive control design (Alexandrov, 1992), stability and controllability analysis and identification (Alexandrov, 1993).

In this paper the identification method that is based on FDP is developed. It is investigated possibility of an identification time reduce.

2. PROBLEM STATEMENT

Consider the plant described by the equation

$$y^{(n)}+d_{n-1}y^{(n-1)}+\ldots+d_1\dot{y}+d_0y=$$
$$=k_\gamma u^{(\gamma)}+\ldots+k_0u+m_\alpha f^{(\alpha)}+\ldots+m_0f \qquad (1)$$

in which $y(t)$ is measured output, $u(t)$ is the controlled input, $f(t)$ is a unknown bounded external disturbance. The coefficients d_i, k_j, m_l ($i=\overline{0,n-1}$, $j=\overline{0,\gamma}$, $l=\overline{0,\alpha}$) are the unknown numbers, n is a given number, $\gamma<n$, $\alpha<n$.

It is assumed that the plant (1) is completely controllable. If the plant is unstable the estimation C_0 of unstability degree s^* is specified ($C_0>s^*$)

$$s^*=\max\{\mathrm{Re}s_1, \ldots, \mathrm{Re}s_n\}, \qquad (2)$$

where s_i ($i=\overline{1,n}$) are the roots of polynomial $d(s)=s^n+d_{n-1}s^{n-1}+\ldots+d_1s+d_0$.

Problem 2.1. Find the estimations of coefficients d_i, k_j ($i=\overline{0,n-1}$, $j=\overline{0,\gamma}$) such that beginning with some moment t^* the following conditions are fulfilled

$$|\hat{d}_j(t)-d_j|\leq\varepsilon^*, \quad |\hat{k}_j(t)-k_j|\leq\varepsilon^*, \quad t\geq t^* \ (i=\overline{0,n-1},\ j=\overline{0,\gamma}), \qquad (3)$$

where ε^* is the given positive number. ■

Time moment t^* is named an identification time.

3. FREQUENCY DOMAIN PARAMETERS (FDP)

Definition 3.1. A set of 2n numbers

$$\alpha_k=\mathrm{Re}W(\lambda+j\omega_k), \quad \beta_k=\mathrm{Im}W(\lambda+j\omega_k), \quad (k=\overline{1,n}),\ \lambda>C_0 \qquad (4)$$

is called the frequency domain parameters. Here $W(s)=k(s)/d(s)$ is the transfer function of the plant, ω_k $(k=\overline{1,n})$ are the positive numbers that are called the test frequencies, $\omega_i\neq 0$, $\omega_i\neq\omega_j$ $(i\neq j)$, $(i=\overline{1,n})$. ∎

The method of the experimental determination of the FDP consists in the following. Apply to the input of the plant (1) the test signal

$$u(t)=e^{\lambda(t-t_0)}\sum_{i=1}^{n}\rho_i\sin\omega_i(t-t_0),\ \lambda>C_0 \quad (5)$$

where λ, ω_i, ρ_i $(i=\overline{1,n})$ are given.

Signal $y(t)$ after multiplication by $e^{-\lambda(t-t_0)}$ is applied to the Fourier filter input. The outputs of the filter give the FDP estimations

$$\alpha_k(t_F,\delta)=\frac{2}{\rho_k\delta}\int_{t_F}^{t_F+\delta}y(t)e^{-\lambda(t-t_0)}\sin\omega_k(t-t_0)dt,$$

$$\beta_k(t_F,\delta)=\frac{2}{\rho_k\delta}\int_{t_F}^{t_F+\delta}y(t)e^{-\lambda(t-t_0)}\cos\omega_k(t-t_0)dt, \quad (6)$$

where $\lambda>C_0$, $k=\overline{1,n}$, t_F is a filtering start time, $t_F\geqslant t_0$.

Theorem 3.1. (Alexandrov, 1993) The FDP estimations have the next property for any bounded external disturbance

$$\lim_{\delta\to\infty}\alpha_k(t_F,\delta)=\alpha_k,\ \lim_{\delta\to\infty}\beta_k(t_F,\delta)=\beta_k\ (k=\overline{1,n}) \quad (7)$$

∎

This property is well known (Eykhoff, 1974) for a stable plant and $f(t)=0$. In fact, if the signal $u(t)=1\cdot\sin\omega_k t$ is applied to the stable plant (1) then its output has the form $y(t)=\alpha_k\sin\omega_k t+\beta_k\cos\omega_k t+\varkappa(t)$, where $\varkappa(t)$ is a vanishing function $(\lim_{t\to\infty}\varkappa(t)=0)$. Fourier filter outputs give α_k and β_k for a sufficiently large filtering time δ. If $f(t)\neq 0$ the described experiment may give the shift of the estimations (for example, if $f(t)=1\cdot\sin\omega_k t$). The use of the test signal (5) with $\lambda>0$ gives the unshift estimations in this case.

4. FREQUENCY EQUATIONS OF IDENTIFICATION

Form Bezout-Identity

$$d(s)\hat{k}(s)-k(s)\hat{d}(s)=s^n k(s) \quad (8)$$

with the unknown polynomials

$$\hat{d}(s)=\hat{d}_{n-1}s^{n-1}+\ldots+\hat{d}_1 s+\hat{d}_0,$$

$$\hat{k}(s)=\hat{k}_\gamma s^\gamma+\hat{k}_{\gamma-1}s^{\gamma-1}+\ldots+\hat{k}_1 s+\hat{k}_0.$$

This identity has an obvious solution

$$\hat{d}(s)=d(s)-s^n=d_{n-1}s^{n-1}+\ldots+d_1 s+d_0,$$

$$\hat{k}(s)=k(s). \quad (9)$$

that is unique (Volovich, 1974) since the polynomials $d(s)$ and $k(s)$ are relatively prime and a degree of polynomial $\hat{d}(s)$ less than degree of polynomial $d(s)$.

Let us divide the identity (8) by the polynomial $d(s)$, place $s=s_k=\lambda+j\omega_k$ $(k=\overline{1,n})$ and obtain

$$\hat{k}(\lambda+j\omega_k)-W(\lambda+j\omega_k)\hat{d}(\lambda+j\omega_k)=(\lambda+j\omega_k)^n W(\lambda+j\omega_k)$$
$$(k=\overline{1,n}) \quad (10)$$

The frequency equations of identification follow from (10) if the expression (4) for the FDP takes into account

$$\sum_{i=0}^{n-1}\rho_i(\omega_k)\hat{k}_i-\sum_{i=0}^{n-1}[\alpha_k\rho_i(\omega_k)-\beta_k\mu_i(\omega_k)]\hat{d}_i=$$
$$=\alpha_k\rho_n(\omega_k)-\beta_k\mu_n(\omega_k),\ (k=\overline{1,n})$$
$$\sum_{i=0}^{n-1}\mu_i(\omega_k)\hat{k}_i-\sum_{i=0}^{n-1}[\alpha_k\mu_i(\omega_k)+\beta_k\rho_i(\omega_k)]\hat{d}_i=$$
$$=\alpha_k\mu_n(\omega_k)+\beta_k\rho_n(\omega_k),\ (k=\overline{1,n}) \quad (11)$$

where $\rho_i(\omega_k)=\mathrm{Re}(\lambda+j\omega_k)^i$, $\mu_i(\omega_k)=\mathrm{Im}(\lambda+j\omega_k)^i$ $(k=\overline{1,n},\ i=\overline{0,n-1})$.

Lemma 4.1. (Alexandrov, 1989) The frequency equations (11) have the unique solution that coincides with the solution (9) of identity (8) if the polynomial $d(s)$ and $k(s)$ are relatively prime and the test frequencies $\omega_k\neq 0$, $\omega_k\neq\omega_i$ $(k\neq i,\ i,k=\overline{1,n})$. ∎

So, the solution of the equations (11) under the arbitrary test frequencies is

$$\hat{d}_i=d_i,\ \hat{k}_j=k_j\ (i=\overline{0,n-1},\ j=\overline{0,\gamma}) \quad (12)$$

To solve the problem 2.1 the FDP estimations $\alpha_k(t)$ and $\beta_k(t)$ $(k=\overline{1,n})$ to be derived on Fourier filter outputs are placed in frequency equations instead of the true FDP and the plant coefficients estimations $\hat{d}_i(t)$, $\hat{k}_j=(t)$ $(i=\overline{0,n-1},\ j=\overline{0,\gamma})$ are the result of this equations solution. In accordance with theorem 3.1 there exists the moment t^* for that the inequalities (3) are fulfilled.

Remark 4.1. The functions $\hat{d}_i(t)$ and $\hat{k}_j=(t)$ $(i=\overline{0,n-1},\ j=\overline{0,\gamma})$ are determined for time moments for that the determinant of system (11) under $\alpha_k=\alpha_k(t)$ and $\beta_k=\beta_k(t)$ $(k=\overline{1,n})$ is not zero. ∎

The identification time t^* depends on the test signal parameters (the numbers ρ_i, ω_i $(i=\overline{1,n})$ and λ), time of the filtering start, the external disturbance level and so on. Further it is investigated the influence each from these factors on identification time.

5. QUADRATURAL AND TEST FREQUENCIES INTERACTION ERRORS

For convenience the plant equation (1) is transformed to Cauchy's form

$$\dot{x}=Ax+bv+\phi f,\ y=d'x,\ x(t_0)=x_0, \tag{13}$$

where $x(t)$ is n-dimensional vector of the plant state, A is a matrix, b, ϕ and d are vectors.

The solution of the differential equation (13) has a view

$$y(t)=y^v(t)+y^o(t)+y^f(t), \tag{14}$$

where

$$y^v(t)=d'\int_{t_0}^{t} e^{A(t-\tau)}be^{\lambda(\tau-t_0)}\times \left[\sum_{i=0}^{n}\rho_i\sin\omega_i(\tau-t_0)\right]d\tau, \tag{15}$$

$$y^o(t)=d'e^{A(t-t_0)}x_0, \tag{16}$$

$$y^f(t)=d'\int_{t_0}^{t} e^{A(t-\tau)}\phi f(\tau)d\tau. \tag{17}$$

The expression (15) may be represented as

$$y^v(t)=y_B^v(t)+æ^v(t), \tag{18}$$

where

$$y_B^v(t)=d'\left[\sum_{i=1}^{n}\alpha^{x}_{(i)}\rho_i\sin\omega_i(t-t_0)+ +\beta^{x}_{(i)}\rho_i\cos\omega_i(t-t_0)\right]e^{\lambda(t-t_0)}, \tag{19}$$

$$æ^v(t)=d'e^{A(t-t_0)}\sum_{i=1}^{n}\rho_i\beta^{x}_{(i)} \tag{20}$$

Here

$$\alpha^{x}_{(k)}=\mathrm{Re}[E(\lambda+j\omega_k)-A]^{-1}b,\ d'\alpha^{x}_{(k)}=\alpha_k,\ (k=\overline{1,n})$$

$$\beta^{x}_{(k)}=\mathrm{Im}[E(\lambda+j\omega_k)-A]^{-1}b,\ d'\beta^{x}_{(k)}=\beta_k, \tag{21}$$

Taking into account the connections (5) and (19) it is easy obtained next expression for the μ-th component of Fourier's filter output that is excited by component $y_B^v(t)$ of the plant output

$$\alpha^{B}_{\mu}(t_F,\delta)=\frac{2}{\rho_\mu\delta}\int_{t_F}^{t_F+\delta}y_B^v(\tau)e^{-\lambda(\tau-t_0)}\sin\omega_\mu(\tau-t_0)d\tau= =\alpha_\mu+e^{\alpha}_{\mu,1}(t_F,\delta)+e^{\alpha}_{\mu,2}(t_F,\delta), \tag{22}$$

where $e^{\alpha}_{\mu,1}(t_F,\delta)$ and $e^{\alpha}_{\mu,2}(t_F,\delta)$ are the quadratural and test frequencies interaction errors respectively

$$e^{\alpha}_{\mu,1}(t_F,\delta)= -\frac{\alpha_k}{2\delta\omega_\mu}[\sin 2\omega_\mu(t_F-t_0+\delta)-\sin 2\omega_\mu(t_F-t_0)]- -\frac{\beta_k}{2\delta\omega_\mu}[\cos 2\omega_\mu(t_F-t_0+\delta)-\cos 2\omega_\mu(t_F-t_0)], \tag{23}$$

$$e^{\alpha}_{\mu,2}(t_F,\delta)= =\frac{2}{\rho_\mu\delta}\int_{t_F}^{t_F+\delta}\sum_{i=1}^{\mu-1}[\alpha_i\rho_i\sin\omega_i(\tau-t_0)\sin\omega_\mu(\tau-t_0)+ +\beta_i\rho_i\cos\omega_i(\tau-t_0)\sin\omega_\mu(\tau-t_0)]d\tau+ +\frac{2}{\rho_\mu\delta}\int_{t_F}^{t_F+\delta}\sum_{i=\mu+1}^{n}[\alpha_i\rho_i\sin\omega_i(\tau-t_0)\sin\omega_\mu(\tau-t_0)+ +\beta_i\rho_i\cos\omega_i(\tau-t_0)\sin\omega_\mu(\tau-t_0)]d\tau. \tag{24}$$

The last errors

$$e^{\alpha}_{i,2}(t_F,\delta)=e^{\beta}_{i,2}(t_F,\delta)=0\ (i=\overline{1,n})$$

if the test frequencies are chosen multiple of some basic frequency ω^*

$$\omega_k=L_k\omega^*\ (k=\overline{1,n}) \tag{25}$$

where L_k $(k=\overline{1,n})$ are some positive integers.

Let $T^*=\frac{2\pi}{\omega^*}$ be a reference period.

The quadratural errors

$$e^{\alpha}_{i,1}(t_F,\delta)=e^{\beta}_{i,1}(t_F,\delta)=0\ (i=\overline{1,n}) \tag{26}$$

if time moments t_F-t_0 and δ are chosen multiple of T^*:

$$t_F-t_0=l\cdot T^*,\ \delta=l_\delta\cdot T^*. \tag{27}$$

Further it is assumed that conditions (25), (27) are fulfilled.

6. FILTERING ERRORS ESTIMATIONS

The filtering error $e^{\alpha}_{\mu}(t_F,\delta)$ arisen because of the components $y^o(t)$, $y^f(t)$ and $æ^v(t)$ of the plant output is

$$e^{\alpha}_{\mu}(t_F,\delta)=e^{\alpha o}_{\mu}(t_F,\delta)+e^{\alpha f}_{\mu}(t_F,\delta)+e^{\alpha æ}_{\mu}(t_F,\delta) \tag{28}$$

where

$$e^{\alpha f}_{\mu}(t_F,\delta)=\frac{2}{\rho_\mu\delta}\int_{t_F}^{t_F+\delta}y^f(\tau)e^{-\lambda(\tau-t_0)}\sin\omega_\mu(\tau-t_0)d\tau \tag{29}$$

$$e^{\alpha o}_{\mu}(t_F,\delta)=\frac{2}{\rho_\mu\delta}\int_{t_F}^{t_F+\delta}y^o(\tau)e^{-\lambda(\tau-t_0)}\sin\omega_\mu(\tau-t_0)d\tau \tag{30}$$

$$e^{\alpha æ}_{\mu}(t_F,\delta)=\frac{2}{\rho_\mu\delta}\int_{t_F}^{t_F+\delta}æ^v(\tau)e^{-\lambda(\tau-t_0)}\sin\omega_\mu(\tau-t_0)d\tau \tag{31}$$

The expressions for errors $e^{\beta o}_{\mu}(t_F,\delta)$, $e^{\beta f}_{\mu}(t_F,\delta)$ and $e^{\beta æ}_{\mu}(t_F,\delta)$ are written in same manner.

Assertion 6.1. The estimations of the filtering errors have a view

$$|e_i^{\alpha f}(t_F,\delta)|\leqslant V_{1,i},\ |e_i^{\beta f}(t_F,\delta)|\leqslant V_{1,i}\quad (i=\overline{1,n}) \tag{32}$$

$$|e_i^{\alpha o}(t_F,\delta)|\leqslant V_{2,i},\ |e_i^{\beta o}(t_F,\delta)|\leqslant V_{2,i}\quad (i=\overline{1,n}) \tag{33}$$

$$|e_i^{\alpha x}(t_F,\delta)|\leqslant V_{3,i},\ |e_i^{\beta x}(t_F,\delta)|\leqslant V_{3,i}\quad (i=\overline{1,n}) \tag{34}$$

where

$$V_{1,i}=\frac{2q_1}{\rho_i\delta s^{**}}\left\{\frac{1}{qe^{q\tau_d}}(1-e^{-q\delta}) - \frac{1}{qe^{q\tau_d}}(1-e^{-\lambda\delta})\right\} \tag{35}$$

$$V_{2,i}=\frac{2q_2}{\rho_i\delta qe^{q\tau_d}}(1-e^{-q\delta}) \tag{36}$$

$$V_{3,i}=\frac{2q_3}{\rho_i\delta qe^{q\tau_d}}(1-e^{-q\delta}) \tag{37}$$

where

$$s^{**}=s^*+\varepsilon_1,\ q=-s^{**}+\lambda,\ (q,\ \lambda>0) \tag{38}$$

ε_1 is a sufficiently small positive number, $\tau_d=t_F-t_0$ is a filtering delay time, q_i (i=1,2,3) are some numbers. ■

The assertion proof is given in Appendix.

The expressions (35)-(37) show that increasing of the delay τ_d up to some bound gives reduce of identification time. This delay is necessary for forming of FDP which are extracted from the plant output in the filtering process and if τ_d is small then the filtering time increases because of the unformed FDP.

Remark 6.1. The theorem 3.1 is a simple consequence of the assertion 5.1. In fact, one easily sees from (35)-(37) that filtering errors estimations have the property of $\lim_{\delta\to\infty} V_{ij}(\delta)=0$ (i=1,2,3; $j=\overline{1,n}$) and therefore the FDP estimations do the property of (7). ■

7. FILTRATION WITH FORGETING

Consider the necessary conditions of convergence of a filtering process

$$\frac{|\alpha_k(t_F,\delta)-\alpha_k(t_F,\delta-\Delta t)|}{|\alpha_k(t_F,\delta)|}\leqslant\varepsilon,\quad \frac{|\beta_k(t_F,\delta)-\beta_k(t_F,\delta-\Delta t)|}{|\beta_k(t_F,\delta)|}\leqslant\varepsilon,\quad (k=\overline{1,n}) \tag{39}$$

where ε is a given sufficiently small positive number, Δt is a positive number that is multiple of T^*

$\Delta t=l_\Delta\cdot T^*$ (l_Δ is a positive integer).

The conditions (39) are often sufficient and that is why they may be used for a determination of identification time finish.

If numbers δ and Δt are fixed and placed $\delta=\delta^*$ and $\Delta t=\Delta t^*$ then there exists a moment t_F such that the conditions (39) is fulfilled.

In fact, in view (22) and (28) the differences in left parts of the inequalities (39) are estimated as

$$\begin{aligned}&|\alpha_k(t_F,\delta^*)-\alpha_k(t_F,\delta^*-\Delta t^*)|\leqslant\\ &\leqslant|e_k^{\alpha f}(t_F,\delta^*)-e_k^{\alpha f}(t_F,\delta^*-\Delta t^*)|+\\ &+|e_k^{\alpha o}(t_F,\delta^*)-e_k^{\alpha o}(t_F,\delta^*-\Delta t^*)|+\\ &+|e_k^{\alpha x}(t_F,\delta^*)-e_k^{\alpha x}(t_F,\delta^*-\Delta t^*)|\quad (k=\overline{1,n})\end{aligned}$$

(in short, here and further the analogous relations for β_k ($k=\overline{1,n}$) are omitted)

Using the estimations (32)-(37) one may concluse that

$$\lim_{t_F\to\infty}|\alpha_k(t_F,\delta^*)-\alpha_k(t_F,\delta^*-\Delta t^*)|=0\quad (k=\overline{1,n}) \tag{40}$$

Now the inequalities (39) will be used for determination of filtering start time t_F.

Denote (under $t_0=0$)

$$\alpha_k(t_F,\delta^*)=\alpha_k(l\cdot T^*,l_\delta^*\cdot T^*)=\alpha_k(l,l_\delta^*) \tag{41}$$

and place $l_\delta^*=2$, $l_\Delta^*=1$.

Form a function

$$\varepsilon_k^\alpha(l)=\frac{|\alpha_k(l,2)-\alpha_k(l,1)|}{|\alpha_k(l,2)|}\quad (k=\overline{1,n}). \tag{42}$$

A positive integer l will be determined from conditions

$$\varepsilon_k^\alpha(l)\leqslant\varepsilon,\ \varepsilon_k^\beta(l)\leqslant\varepsilon\quad (k=\overline{1,n},\ l=1,2,\ldots) \tag{43}$$

Transform the expressions (42) to more simple view. Denote to this effect

$$J_{k,i}^\alpha = \int_{(i-1)T^*}^{iT^*} y(\tau)e^{-\lambda(\tau-t_0)}\sin\omega_k(\tau-t_0)d\tau \quad (k=\overline{1,n},\ l=1,2,\ldots) \tag{44}$$

Then the relations (6) and (42) are rewritten as

$$\alpha_k(l,l_\delta)=\frac{2}{\rho_k l_\delta T^*}\sum_{i=l+1}^{l+l_\delta+1} J_{k,i}^\alpha\quad (k=\overline{1,n}) \tag{45}$$

$$\varepsilon_k^\alpha(l)=\frac{J_{k,l+2}^\alpha-J_{k,l+1}^\alpha}{J_{k,l+2}^\alpha+J_{k,l+1}^\alpha}\quad (k=\overline{1,n},\ l=1,2,.) \tag{46}$$

Consider the next integral to find the numbers $J_{k,l+1}^\alpha$ and $J_{k,l+2}^\alpha$ in the real time

$$\int_0^{(l+2)T^*} y(\tau)e^{-\lambda(\tau-t_0)}\sin\omega_k(\tau-t_0)d\tau= =J_{k,1}^\alpha+J_{k,2}^\alpha+J_{k,3}^\alpha+J_{k,4}^\alpha+\ldots+J_{k,l+2}^\alpha. \tag{47}$$

For calculation $\varepsilon_k^{\alpha}(l)$ two last summand of the sum are used but other its components are forgotten.

Remark 7.1. Instead of (39) the following analogous inequalities may be used as the necessary conditions

$$\frac{|\hat{d}_i(t_F,\delta)-\hat{d}_i(t_F,\delta-\Delta t)|}{|\hat{d}_i(t_F,\delta)|}\leqslant\varepsilon_o,$$
$$\frac{|\hat{k}_i(t_F,\delta)-\hat{k}_i(t_F,\delta-\Delta t)|}{|\hat{k}_i(t_F,\delta)|}\leqslant\varepsilon_o, \quad (i=\overline{0,n-1}) \quad (48)$$

where $\hat{d}_i(t_F,\delta)$, $\hat{k}_i(t_F,\delta)$ and $\hat{d}_i(t_F,\delta-\Delta t)$, $\hat{k}_i(t_F,\delta-\Delta t)$ $(i=\overline{1,n-1})$ are the solutions of the frequency equations (11) under $\alpha_k=\alpha_k(t_F,\delta)$, $\beta_k=\beta_k(t_F,\delta)$ and $\alpha_k=\alpha_k(t_F,\delta-\Delta t)$, $\beta_k=\beta_k(t_F,\delta-\Delta t)$ $(k=\overline{1,n})$ respectively; ε_o is a positive number characterized an identification accuracy (usually, $0.1\leqslant\leqslant\varepsilon_o\leqslant0.01$).

If the inequalities (39) are sufficient for a filtering process convergence then from (7) and continuity of a solution of the linear equation (11) it follows

$$\lim_{\delta\to\infty}\hat{d}_i(t_F,\delta)=d_i, \quad \lim_{\delta\to\infty}\hat{k}_i(t_F,\delta)=k_i, \quad (i=\overline{0,n-1}) \quad (49)$$

To find a filtering start time for that the conditions (48) are fulfilled it need to solve the frequency equation (11) and to check the condition (48) for each l (l=1,2,...). ■

8. EXAMPLE

Consider the plant described by the equation (Fomin, *et al*, 1981)

$$\ddot{y}+d_1\dot{y}+d_0y=k_1\dot{u}+k_0u+f, \quad t_0=0 \quad (50)$$

with the unknown coefficients d_1, d_0, k_1, k_0.

Let us be known that the plant is completely controllable and the estimation of its unstability degree $C_0=4.5$.

Problem 8.1. Find the estimations $\hat{d}_1(t^*)$, $\hat{d}_0(t^*)$, $\hat{k}_1(t^*)$, $\hat{k}_0(t^*)$ that satisfy inequalities (3) under $n=2$, $\gamma=1$, $\varepsilon^*=1$. ■

Remark 8.1. The true coefficients of plant

$$d_0=-16,\ d_1=0,\ k_0=30,\ k_1=5, \quad (51)$$

and the external disturbance $f(t)=\sin1.5t$. ■

The numerical experiments have been performed under the test signal

$$u(t)=0.01e^{5t}(\sin3t+\sin6t)$$

and different numbers ε in the inequalities (43).

In the first experiment it was placed $\varepsilon=0.1$ and obtained

$l=2 \quad (t^*=8.38$ s.)

$$\hat{d}_0=-13.1,\ \hat{d}_1=-0.67,\ \hat{k}_0=25.5,\ \hat{k}_1=5.2 \quad (52)$$

In the second experiment $\varepsilon=0.01$ and it was obtained

$l=3 \quad (t^*=10.4$ s.)

$$\hat{d}_0=-15.6,\ \hat{d}_1=-0.085,\ \hat{k}_0=29.4,\ \hat{k}_1=5.01 \quad (53)$$

It easy sees that last estimations satisfy the requirement (3) on accuracy of identification.

9. CONCLUSION

The procedure of a linear continuous plant identification is developed. It bases self upon FDP estimations and the frequency equation solution. Convergence of this procedure for any bounded external disturbance is proved .

The filtering errors estimations that show significance of the filtering delay time are obtained. Algorithm of the filtration with forgeting allowed to decrease an identification time is proposed.

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APPENDIX

The proof of the assertion 6.1

At first find the estimations of the filtering errors because of the external disturbance bounded by a number f^* ($|f(t)|\leqslant f^*$).

Denote

$$h(t-\tau)=d'e^{A(t-\tau)}\phi$$

and rewrite (17) as

$$y^f(t)=\int_{t_0}^{t} h(t-\tau)f(\tau)d\tau. \qquad (A.1)$$

It is obvious

$$|y^f(t)|\leqslant f^*\int_{t_0}^{t} |h(t-\tau)|d\tau. \qquad (A.2)$$

It is known (Demidovich, 1967) that

$$|h(t-\tau)|\leqslant\tilde{q}_1 e^{s^{**}(t-\tau)}. \qquad (A.3)$$

where $\tilde{q}_1$ is a number.

Now the expression (A.2) has a view

$$|y^f(t)|\leqslant f^*\tilde{q}_1\int_{t_0}^{t} e^{s^{**}(t-\tau)}d\tau=$$

$$=\frac{f^*\tilde{q}_1}{s^*}[e^{s^{**}(t-t_0)}-1]. \qquad (A.4)$$

In other hand the error estimation follows from (30)

$$|e_\mu^{\alpha f}(\delta)|\leqslant\frac{2}{\rho_\mu\delta}\int_{t_F}^{t_F+\delta} |y^f(\tau)|e^{-\lambda(\tau-t_0)}\times \qquad (A.5)$$

$$\times|\sin\omega_\mu(\tau-t_0)|d\tau\leqslant\frac{2}{\rho_\mu\delta}\int_{t_F}^{t_F+\delta} |y^f(\tau)|e^{-\lambda(\tau-t_0)}d\tau$$

The connection (35) under $i=\mu$ is obtained if inequality (A.4) is substituted into (A.5). Described is true for any μ ($\mu=\overline{1,n}$) and may be repeated for the error $e_\mu^\beta(\delta)$.

To prove the inequalities (33) it is denoted $h_0(t-\tau)=d'e^{A(t-\tau)}x_0$ and obtained the estimation analogous (A.3):

$$|y^0(t)|=|h_0(t-t_0)|\leqslant\tilde{q}_2 e^{s^{**}(t-t_0)}, \qquad (A.6)$$

that is substituted into the error estimation

$$|e_\mu^{\alpha 0}(\delta)|\leqslant\frac{2}{\rho_\mu\delta}\int_{t_F}^{t_F+\delta} |y^0(\tau)|e^{-\lambda(\tau-t_0)}\, d\tau.$$

The inequalities (34) are proved analogously.