# FINITE-FREQUENCY CRITERIONS OF STABILITY OF AUTOMATIC CONTROL SYSTEMS WITH UNCERTAIN PARAMETERS.

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**Abstract.** Two new criterions of the stability of the system accessible for the experimental investigation is proposed. The criterions take precedence over the Nyquist criterion: a) the finite number of the test frequencies, b) the information about the open-loop system roots with the positive real parts which is required for the proposed criterions can be obtained from the experiments.

**Keywords.** Stability, uncertain parameters, frequency domain, experimental investigations, criterions of stability.

## 1. INTRODUCTION

One of the basic problems of the theory of the control systems with the uncertain parameters is the problem of stability analysis. We can distinguinsh three directions in the solution of this problem.

First direction. It is supposed that lower and upper bounds of the system parameters are known (The structured uncertain systems). For this case the sufficient conditions of stability have been given in the literature which is began with [1]; also recent contributions such as for example [2] can be found.

The second direction. The model of the plant is characterised by a given fixed nominal plant with a set of norm-bounded perturbation (The unstructured uncertain system). Then the conditions required to the robust disturbance rejection is derived [3],[4].

The trird direction have been began by Nyquist [5]. It is supposed that the plant (or open-loop system) is accessible for the experimental investigations. For using of the Nyquist criterion is necassary the frequency response of open-loop system in the frequency bandwidth from zero to infinity [6].

This infinite series of the experiments may be decreased if the parameter of the transfer function of open-loop systems is identifiedon the basis of the test from the finite number of the frequency [7] and then the frequency responses in the all frequency bandwidth can be calculated.

This is an obvious way to decrease of the number of the experiments. Is there an other way? This way is to construct the direct stability criterion, which is based on the finite number of the experiments and in which the identification of the parameters is not used and the frequency responses is not calculated.

Such direct criterion has been derived in [8]. It to base self upon the solution of the inverse problem of the optimal control [9] and the solution of LQG-problem [10] for the plants specified by finite numbers of values of their frequency response [11].

The aim of this paper is the construction of the new direct stability criterion based self upon the modal approach.

## 2. PRELIMINARIES

### 2.1. The frequency domain parameters and theirs experimental determination.

Consider the plant described by the following equation

$$(s^n+d_{n-1}s^{n-1}+\dots+d_0)y=(k_\gamma s^\gamma+\dots+k_0)u, \quad \gamma<n \qquad (1)$$

where y(t) is the measured output, u(t) is the control input.

The transfer function of the plant (1) is

$$w(s)=k(s)/d(s).$$

Let $C_0$ is the estimation of an upper boundary of the set of the positive numbers which are the values of the real part roots of the polynomials d(s). If the roots of d(s) lie in left half-plan, then $C_0=0$.

Definition 2.1 The set of the 2n numbers

$$\alpha_k=\mathrm{Re}\,w(\lambda+j\omega_k), \quad \beta_k=\mathrm{Jm}\,w(\lambda+j\omega_k), \quad (k=\overline{1,n}) \qquad (2)$$

(where $\omega_k(k=\overline{1,n})$ are some positive numbers, $\lambda\ge C_0$) are called the frequency domain parameters of the plant (1). □

The method of the experimental determination of the frequency domain parameters is well known [7], for a stable plant.

In fact if we apply to the input of the plant (1) the harmonic signal $u(t)=1\cdot\sin\omega t$, then we obtain on its output

$$y(t)=\alpha_k\sin\omega_k t+\beta_k\cos\omega_k t+x(t), \qquad (3)$$

where x(t) is the vanich function $(\lim_{t\to\infty} x(t)=0)$. Applying the signal (3) to the Fourier filter we can receive the number $\alpha_k$ and $\beta_k$.

In case of the unstable plant (1), we have to apply the signal

$$u(t)=e^{\lambda t}\sin\omega_k t \qquad (\lambda\ge C_0)$$

to the input of the plant (1) and then apply the signal $y(t)e^{-\lambda t}$ on the input of the Fourier filter [12].

Definition 2.2 The numbers $\omega_k$ $(k=\overline{1,n})$ in (2) are called the test frequencies. □

Definition 2.3 The set of the p numbers (2), where $n<p\le 2n$ are called the widened frequency domain parameters of the plant (1). □

Remark 2.1 The frequency domain parameters exist if the test frequencies satisfy the conditions

$$d(\lambda+j\omega_k)\ne 0 \quad (k=\overline{1,n} \text{ or } k=\overline{1,p}). \qquad (4)$$

### 2.2 Bezout-Identity

Consider the Bezout-Identity

$$\gamma_2(s)\delta_2(s)-\gamma_1(s)\delta_1(s)=\gamma_3(s) \qquad (5)$$

in which $\gamma_1(s)$, $\gamma_2(s)$ and $\gamma_3(s)$ are the given polynomials. These polynomials have the following degrees

$$\partial[\gamma_i(s)]=\rho_i \quad (i=1,2,3). \qquad (6)$$

Theorem 2.1 If the polynomials $\gamma_1(s)$ and $\gamma_2(s)$ are relatively prime then polynomials $\delta_1(s)$ and $\delta_2(s)$ exist.

If moreover

$$\partial[\delta_2(s)]<\rho_1 \quad \text{or} \quad \partial[\delta_1(s)]<\rho_2, \qquad (7)$$

then this pair of the polynomials is unique. □

The proof of this theorem is contained in [13], [14].

Let

a) $\gamma_3(s)\equiv 0$ (8)

b) $\partial[\delta_2(s)]=\rho_1,\ \partial[\delta_1(s)]=\rho_2.$ (9)

c) $\delta_{1\rho_1}=\gamma_{2\rho_2}$ (10)

where $\delta_{1\rho_1}$ and $\gamma_{2\rho_2}$ are the coefficients by the highest degrees of polynomials $\delta_1(s)$ and $\gamma_2(s)$ respectively.

Lemma 2.1 If the polynomials $\gamma_1(s)$ and $\gamma_2(s)$ are relatively prime and the conditions (9), (10) are fulfilled then the identity

$$\gamma_2(s)\delta_2(s)-\gamma_1(s)\delta_1(s)=0 \qquad (11)$$

has the obvious solution

$$\delta_2(s)=\gamma_1(s), \quad \delta_1(s)=\gamma_2(s) \qquad (12)$$

which is unique. □

Proof. Let there is another than (12) solution of the (11):

$$\delta_2(s)=\delta_2^*(s), \quad \delta_1(s)=\delta_1^*(s) \qquad (13)$$

which satisfies the conditions (9) and (10).

Then

$$\gamma_2(s)\delta_2^*(s)-\gamma_1(s)\delta_1^*(s)=0.$$

Substracting this identity from (11) we obtain

$$\gamma_2(s)\bar{\delta}_2(s)-\gamma_1(s)\bar{\delta}_1(s)=0, \qquad (14)$$

where $\bar{\delta}_2(s)=\gamma_1(s)-\delta_2^*(s)$, $\bar{\delta}_1(s)=\gamma_2(s)-\delta_1^*(s)=(\gamma_{2,\rho_2-1}-\delta^*_{1,\rho_2-1})s^{\rho_2-1}+\ldots+(\gamma_{2,0}-\delta^*_{1,0})$.

Since $\partial[\bar{\delta}_1(s)]=\rho_2-1<\rho_2$ then on basis of theorem 2.1 we conclude that the identity (14) has the unique solution

$$\bar{\delta}_1(s)=\bar{\delta}_2(s)\equiv 0 \qquad (15)$$

# 3. PROBLEM STATEMENT

## 3.1. Problem of the systems stability with the known controllers

Consider the control system which consist of the plant (1) and the controller

$$(s^{n-1}+g_{n-2}s^{n-2}+\ldots+g_1s+g_0)u=(r_{n-1}s^{n-1}+\ldots+r_1s+r_0)y \qquad (16)$$

Make the following assumptotions.

A1. The coefficients $g_i$, $r_j$ $(i,j=\overline{0,n-1})$ of the controller (16) are known.

A2. The coefficients $d_i(i=\overline{0,n-1})$ and $k_j$ $(j=\overline{0,\gamma})$ of the plant (1) are unknown.

A3. The degrees n and $\gamma$ in the equation (1) are the given numbers. For simplicity we assume that $\gamma$ is the even number.

A4. The polynomials d(s)g(s) and k(s)r(s) are relative prime.

A5. For the unstable plant (1) knows the estimation $C_0$ of the real parts of the polynomials d(s) roots.

A6. The plant (1) is accessible for the experimental investigations and its widened frequency domain parameters $\alpha_k$ and $\beta_k$ $(k=1,p,\ p=(n+\gamma)/2)$ are the result of this investigations.

Problem 3.1 Find the conditions of the stability of the system (1), (16) which satisfies assumptions A1-A6. □

## 3.2. Problem of the system stability with the unknown controllers

Let the coefficients of the controller (16) are unknown. Then we substitute for A1-A5 by the corresponded assumptions.

A1°. The coefficients $g_i$, $r_j$ $(i,j=\overline{0,n-1})$ of controller (16) are unknown.

A2°. The coefficients $d_i(i=0,\overline{n-1})$, $k_j$ $(j=\overline{0,\gamma})$ of plant (1) are unknown.

A3°. The degrees n and $\gamma$ of plant (1) are the given numbers.

A4°. The polynomials d(s)g(s) and k(s)r(s) are relative prime.

A5°. If the open-loop system (1), (16) is unstable then the estimation $C_{00}$ of the real parts of the polynomial d(s)g(s) roots is known.

A6°. The open-loop system (1), (16) is accesible for the experimental investigations.

he frequency domain parameters

$$\alpha_{0k}=\text{Re}w_0(\lambda+j\omega_k),\quad \beta_{0k}=\text{Jm}w_0(\lambda+j\omega_k),\quad )k=1,\rho_0) \qquad (17)$$

(where

$$w_0(s)=-\frac{k(s)r(s)}{d(s)g(s)},\quad \rho_0=\frac{3n+\gamma-1}{2} \qquad (18)$$

$3n+\gamma-1$ is even number) are the result of this investigations.

Problem 3.2 Find the conditions of stability of the system (1),(16) wich satisfy the assumptions A1°-A6°.

## 3.3 Comment

Remark 3.1 The assumptions A3 and A3° may be cancelled. This isdiscussed in section 5. □

Remark 3.2 The assumptions A4 and A4° is usualy. As early as Nyquist stability criterion the assumptions A4 and A4° are usualy adopted implicity at least for the unstable open-loop system. □

Remark 3.3 The assumptions A5 (and A5°) may be canceled also, if we complicates of the experiments which are used for finding of the frequency domain parameters.

In fact we apply to the input of plant (1) the signal $u=e^{\lambda_1 t}\cdot\sin\omega_k t$ where $\lambda_1$ is some positive number.

If the product $y(t)e^{\lambda_1 t}$ increases then we have to apply the signal $u(t)=e^{\lambda_2 t}\cdot\sin\omega_k t$ (where $\lambda_2>\lambda_1$) and to repeat the experiment till the number $\lambda$ for which the product $y(t)e^{\lambda t}$ is the bounded function will be obtained. □

# 4. MAIN RESULTS (SOLUTION OF PROBLEM 3.1)

## 4.1 Preliminary transformatios (analysis of the stability for the known parameters of the plant)

Let the coefficients of the system (1), (16) knows. The known algorithm of analysis of the system (1), (16) stability may be formulated as follows.

**Algorithm 4.1**

**Step 1.** Form the characteristic polynomial of the system (1), (16)

$$a(s)=d(s)g(s)-k(s)r(s) \tag{19}$$

**Step 2.** *Compose the Hurwitz determinants using the* coefficients of the polynomial (19).

**Step 3.** Check the signs of the Hurwitz determinants. If these signs are positive then the system (1), (16) is stable (asymptotically stable).

We substitute for the expression (19) by the identity

$$a(s)\delta_2(s)-k(s)\delta_1(s)=0 \tag{20}$$

where $a(s)$ and $k(s)$ are the given polynomials of the degrees $\rho_2=2n-1$ and $\rho_1=\gamma$; $\delta_1(s)$ and $\delta_2(s)$ are some polynomials of the degrees $2n-1$ and $\gamma$ respectively.

Assertion 4.1 If the polynomials $a(s)$ and $k(s)$ are relatively prime and $\delta_{1,2n-1}=1$ then the identity (20) has of the unique solution

$$\delta_1(s)=a(s), \quad \delta_2(s)=k(s). \tag{21}$$

□

The proof of this assertion follows from the lemma 2.1.

Assertion 4.2 If the assumption A4 is fulfilled then the polynomials $a(s)$ and $k(s)$ are relatively prime. □

Prooof. Let the polynomials $a(s)$ and $k(s)$ have the common divisor $\nu(s)\neq\text{const}$. Then $a(s)=\bar{a}(s)\nu(s)$, $k(s)=\bar{k}(s)\nu(s)$ where polynomials $\nu(s)$ is some polynomial of the degree $\mu\leq\gamma$.

The expression (19) may be rewritten as

$$d(s)g(s)=[\bar{a}(s)+r(s)\bar{k}(s)]\nu(s).$$

Hence the polynomials $d(s)g(s)$ and $k(s)r(s)$ have the common divisor $\nu(s)$. This contradicts to the assumption A4.

### 4.2 Expression of the identity (20) through the frequency domain parameters of the plant (1)

We divide (20) by the polynomial $d(s)$ and taking into account (19) obtain

$$[g(s)-r(s)w(s)]\delta_2(s)-w(s)\delta_1(s)=0 \tag{22}$$

This identity implies the following system of the equations

$$\{g(\lambda+j\omega_k)-r(\lambda+j\omega_k)[\alpha_k+j\beta_k]\}\delta_2(\lambda+j\omega_k)-[\alpha_k+j\beta_k]\times$$
$$\times\,\delta_1(\lambda+j\omega_k)=0, \quad (k=\overline{1,p}) \tag{23}$$

which may be rewritten as

$$\sum_{i=0}^{\gamma}(\lambda+j\omega_k)^i(l_k+jm_k)\delta_{2,i}-\sum_{i=0}^{2n-1}(\lambda+j\omega_k)^i\times$$
$$\times\,(\alpha_k+j\beta_k)\delta_{1,i}=0, \qquad (k=\overline{1,p}) \tag{24}$$

where

$$l_k=\mathrm{Re}[g(\lambda+j\omega_k)-r(\lambda+j\omega_k)(\alpha_k+j\beta_k)], \tag{25}$$
$$m_k=\mathrm{Im}[g(\lambda+j\omega_k)-r(\lambda+j\omega_k)(\alpha_k+j\beta_k)]. \tag{26}$$

Find

$$(\lambda+j\omega_k)^i=\rho_i(\omega_k)+j\mu_i(\omega_k) \quad (i=\overline{1,2n-1}) \tag{27}$$

where

$$\rho_i(\omega_k)=\sum_{\nu=0}^{[i/2]}q^i_{2\nu}\omega_k^{2\nu}, \quad \mu_i(\omega_k)=\sum_{\nu=0}^{[i/2]}q^i_{2\nu+1}\omega_k^{2\nu+1}, \tag{28}$$

The coefficients $q^i_{2\nu}$ and $q^i_{2\nu+1}$ $(i=\overline{1,2n-1},\ \nu=0,[i/2])$ are the known functions of $\lambda$. $[i/2]$ is the whole part of $i/2$.

### 4.3 The first criterion of stability

We substitute the expression (27) in (25), (26) and taking into account that in (20)

$$\delta_{1,2n-1}=1 \tag{29}$$

obtain the system

$$\sum_{i=0}^{\gamma}[l_k\rho_i(\omega_k)-m_k\mu_i(\omega_k)]\delta_{2,i}-\sum_{i=0}^{2n-2}[\alpha_k\rho_i(\omega_k)-\beta_k\mu_i(\omega_k)]\delta_{1,i}=$$
$$=\alpha_k\rho_{2n-1}(\omega_k)-\beta_k\mu_{2n-1}(\omega_k) \qquad (k=\overline{1,p}), \tag{30}$$

$$\sum_{i=0}^{\gamma}[l_k\mu_i(\omega_k)+m_k\rho_i(\omega_k)]\delta_{2,i}-\sum_{i=0}^{2n-2}[\alpha_k\mu_i(\omega_k)+\beta_k\rho_i(\omega_k)]\delta_{1,i}=$$
$$=\alpha_k\mu_{2n-1}(\omega_k)+\beta_k\rho_{2n-1}(\omega_k)=0 \qquad (k=\overline{1,p}) \tag{31}$$

in which

$$l_k=\sum_{i=0}^{n-1}\rho_i(\omega_k)g_i-\sum_{i=0}^{n-1}[\alpha_k\rho_i(\omega_k)-\beta_k\mu_i(\omega_k)]r_i \quad (k=\overline{1,p}) \tag{32}$$

$$m_k=\sum_{i=0}^{n-1}\mu_i(\omega_k)g_i-\sum_{i=0}^{n-1}[\alpha_k\mu_i(\omega_k)+\beta_k\rho_i(\omega_k)]r_i \quad (k=\overline{1,p}). \tag{33}$$

The system (31), (32) consists of $2n+\gamma$ the linear algebraic equations for the same number of the unknown $\delta_{1,2n-2}, \delta_{1,1}, \delta_{1,0}, \delta_{2,\gamma},\dots,\delta_{2,0}$ calculation.

Theorem 4.1 If the assumption A4 is fulfilled then the system (30),(31)
(a) has the solution,
(b) this solution is unique for arbitrary $\lambda$ and $\omega_k$ $(k=\overline{1,n})$,
(c) $\delta_{1j}$ and $\delta_{2i}$ $(i=\overline{0,\gamma},\ j=\overline{0,2n-1})$ have the values for which the equality (20) is fulfilled.

The property (c) signifies that the solution of the system (30), (31) gives the coefficients of the characteristic polynomial of the system (1),(16).

□

The proof is given by Appendix A.

Criterion 4.1 (The first criterion of stability).

Let the assumptions A1-A6 are fulfilled then the system (1), (16) is asimptotically stable if and only if the solutions $\delta_{1,i}$ $(i=\overline{0,2n-2})$ of the system (30), (31) satisfy the inequalities

$$\Delta_i>0 \quad (i=\overline{1,2n-1}) \tag{34}$$

where $\Delta_i$ $(i=1,\overline{2n-1})$ are the Hurwitz determinants:

$$\Delta_1=\delta_{1,2n-2},\ \Delta_2=\det\begin{bmatrix}\delta_{1,2n-2} & \delta_{1,2n-4}\\ 1 & \delta_{1,2n-3}\end{bmatrix},\dots,\Delta_{2n-1}. \tag{35}$$

□

### 4.4 Example

Consider the plant (the cycle [13]) described by the equation

$$\ddot{y}+d_0y=k_1\dot{u}+k_0 \tag{36}$$

with the controller (the robotcyclist)

$$\dot{u}+8u=-3\dot{y}-13y \tag{37}$$

The coefficients $d_0$, $k_1$, and $k_0$ of the plant is unknown. The widened frequency domain parameters of the cycle are

$$\alpha_1=0.85,\ \beta_1=-1.4,\ \alpha_2=0.22,\ \beta_2=-0.88,\ \alpha_3=0.078,\ \beta_3=-0.6 \tag{38}$$

This values was derived «as the results of the experimental investigations of cycle» for

$$\lambda=6,\ \omega_1=3,\ \omega_2=6,\ \omega_3=9, \tag{39}$$

We shall be investigate the stability of system (36), (37) using of the criterion 4.1. In considered case the equation (23) have the view

$$\{g_0+(\lambda+j\omega_k)g_1-[r_0+r_1(\lambda+j\omega_k)][\alpha_k+j\beta_k]\}\{\delta_{2,1}(\lambda+j\omega_k)+$$
$$+\delta_{2,0}\}-[\alpha_k+j\beta_k]\{(\lambda+j\omega_k)^3+\delta_{1,2}(\lambda+j\omega_k)^2+\delta_{1,1}(\lambda+j\omega_k)+$$
$$+\delta_{1,0}\}=0, \quad (k=1,2,3) \tag{40}$$

where $g_0=8$, $g_1=1$, $r_0=-13$, $r_1=-3$. The system of equations in the form (30), (31) follows from (40)

$$(l_k\lambda-m_k\omega_k)\delta_{2,1}+l_k\delta_{2,0}-[\alpha_k(\lambda^2-\omega_k^2)-\beta_k 2\lambda\omega_k]\delta_{1,2}-(\alpha_k\lambda- -\beta_k\omega_k)\delta_{1,1}-\alpha_k\delta_{1,0}=\alpha_k(\lambda^3-3\lambda\omega_k^2)-\beta_k(-\omega_k^3+3\lambda^2\omega_k),\ (k=\overline{1,3}) \quad (41)$$

$$(l_k\omega_k+m_k\lambda)\delta_{2,1}+m_k\delta_{2,0}-[\beta_k(\lambda^2-\omega_k^2)+\alpha_k 2\lambda\omega_k]\delta_{1,2}-(\alpha_k\omega_k+ +\beta_k\lambda)\delta_{1,1}-\beta_k\delta_{1,0}=\beta_k(\lambda^3-3\lambda\omega_k^2)+\alpha_k(-\omega_k^3+3\lambda^2\omega_k),\ (k=\overline{1,3}) \quad (42)$$

where

$$l_k=g_0+g_1\lambda-r_0\alpha_k-r_1(\lambda\alpha_k-\omega_k\beta_k),\quad (k=1,3) \quad (43)$$

$$m_k=g_1\omega_k-r_0\beta_k-r_1(\omega_k\alpha_k+\lambda\beta_k)\quad (k=\overline{1,3}) \quad (44)$$

From the formulas (43), (44)

$$a_1=52.9,\ a_2=36.7,\ a_3=32.6,\ k_1=-33,\ k_2=-17.4,\ k_3=-7.5 \quad (45)$$

Substituting this numbers in (41), (42) and solwing this system we obtain

$$\delta_{2,1}=5,\ \delta_{2,0}=30,\ \delta_{1,2}=23,\ \delta_{1,1}=139,\ \delta_{1,0}=262 \quad (46)$$

Since $\delta_{1,i}>0$ (i=0,1,2) and $\delta_{1,2}\delta_{1,1}>\delta_{1,0}$ then we conclude that the cycle with the robot-cyclist is asimptotically stable.

Remark 4.3 The frequency domain parameters (38) was derived in result of the simulation runs with the equation (36) with the coefficients $d_0=-16$, $k_0=30$, $k_1=5$.

The characteristic polynomial of the system (36), (37) with this coefficients is written as

$$a(s)=s^3+23s^2+139s+262. \quad (47)$$

Obvious that the coefficients (47) coincide with (46).

□

## 5. Second kriterion of stability (solution of problem 3.2)

### 5.1 Second way of the transformations (19)

Let the coefficients (1),(16) are known. Substitute the expression (19) by the identity

$$a(s)\delta_2(s)-b(s)\delta_1(s)=0 \quad (48)$$

where a(s) and

$$b(s)=k(s)r(s) \quad (49)$$

are the given polynomials of the degrees 2n-1 and n+γ-1 respectively.

Assertion 5.1 If the polynomials a(s) and b(s) are relatively prime and $\delta_{1,2n-1}=1$ then the identity (48) has the unique solution

$$\delta_1(s)=a(s),\quad \delta_2(s)=b(s) \quad (50)$$

□

The proof of this assertion follows from the lemma 2.1.

Assertion 5.2 If the assumption $A4^0$ is fulfilled then a(s) and b(s) are relatively prime.

□

The proof of this assertion repeats of the proof of the assertion 4.2.

### 5.2 Expression of the identity (48) through the frequency domain parameters of the open-loop system

Divide (48) by the polynomial d(s)g(s) and taking into account (19) we obtain

$$[1+w_0(s)]\delta_2(s)+w_0(s)\delta_1(s)=0 \quad (51)$$

The system of the equations follows from (51)

$$(1+\alpha_{0k}+j\beta_{0k})\delta_2(\lambda+j\omega_k)+(\alpha_{0k}+j\beta_{0k})\delta_1(\lambda+j\omega_k)=0,\ (k=\overline{1,p_0}) \quad (52)$$

It may be rewritten by $\delta_{1,2n-1}=1$ as

$$\sum_{i=0}^{n+\gamma-1}[(1+\alpha_{0k})\rho_i(\omega_k)-\beta_{0k}\mu_i(\omega_k)]\delta_{2,i}-\sum_{i=0}^{2n-2}[\alpha_{0k}\rho_i(\omega_k)-\beta_{0k}\times \times\mu_i(\omega_k)]\delta_{1,i}=-[\alpha_{0k}\rho_{2n-1}(\omega_k)+\beta_{0k}\mu_{2n-1}(\omega_k)],\ (k=\overline{1,p_0}) \quad (53)$$

$$\sum_{i=0}^{n+\gamma-1}[(1+\alpha_{0k})\mu_i(\omega_k)+\beta_{0k}\rho_i(\omega_k)]\delta_{2,i}+\sum_{i=0}^{2n-2}[\alpha_{0k}\mu_i(\omega_k)+\beta_{0k}\times \times\rho_i(\omega_k)]\delta_{1,i}=-[\alpha_{0k}\mu_{2n-1}(\omega_k)+\beta_{0k}\rho_{2n-1}(\omega_k)],\ (k=\overline{1,p_0}) \quad (54)$$

### 5.3 The second criterion of stabibility

Criterion 5.1 The system (1),(16) for which the assumptions $A1^0$-$A6^0$ are fulfilled, is the asimptotically stable if and only if the solutions $\delta_{1,i}$ $(i=\overline{0,2n-2})$ of system (53), (54) satisfies of the inequalities (34).

### 5.4 Analysis of stability for the unknown n and γ

Let the degrees n and γ in the equation in the plant (1) is not known.

Then the following procedure may be supposed.

Procedure 5.1

S t e p 1. Assign some number n (Let for simplity $n_1=1$).

S t e p 2. Find the $2n_1$ of the frequency domain parameters of the open-loop system (1), (16) using an experiment.

S t e p 3. Solve the system (53), (54) wich $n=n_1$, $\gamma=n_1-1$ and form the polynomial

$$\delta_1(s)=s^{2n_1-1}+\delta_{1,2n-2}s^{2n_1-2}+\ldots+\delta_{1,0}. \quad (55)$$

S t e p 4. Calculate the roots $\lambda_{1,n_1}\le\lambda_{2,n_1}\le\ldots\le\lambda_{2n_1-1,n_1}$ of this polynomial.

S t e p 5. Return to the step 1 replacing $n_1$ by $n_2=n_1+1$. Repeating the steps 2-4 by $n_2$ obtain the roots $\lambda_{1,n_2}\le\lambda_{2,n_2}\le\ldots\le\lambda_{2n_2-1,n_2}$.

S t e p 6. Check the inequality

$$|\lambda_{i,n_1}-\lambda_{i,n_2}|\le\varepsilon\ (i=1,2n_1-1) \quad (56)$$

$$|\lambda_{2n_1-1,n_1}-\lambda_{2n_2-2,n_2}|\ge\eta_1,\ |\lambda_{2n_1-1,n_1}-\lambda_{2n_2-1,n_2}|\ge\eta_2 \quad (57)$$

where ε are the sufficiently small number, $\eta_1$ and $\eta_2$ are the sufficiently big numbers. If the inequality (56), (57) is fulfilled then go to the step 7. If the one (or several) of the inequality (56), (57) is violated then go to the step 1 replacing $n_2$ by $n_3=n_2+1$ and so on.

S t e p 7. Check the signs of the real part roots $\lambda_{i,n_j}$ $(i=\overline{1,2n_j-1})$ where j (j≥2) is the number of the step of the iterative process for which the follows inequality is fullfilled

$$|\lambda_{i,n_{j-1}}-\lambda_{i,n_j}|\le\varepsilon\quad (i=1,2n_{j-1}-1) \quad (58)$$

$$|\lambda_{2n_{j-1}-1,n_{j,1}}-\lambda_{2n_j-2,n_j}|\ge\eta_1,\ |\lambda_{2n_{j-1},n_{j-1}}-\lambda_{2n_j-1,n_j}|\ge\eta_2 \quad (59)$$

## 6. Conclusion

Two new criterions of the stability has been proposed. The distinctions between these criterions and the Nyquist criterion are the follows.

1. The number of the test frequences is less than or equal to 2n. For the Nyquist criterion this number is infinite.

2. The estimation of the upper boundary of the set of the real parts of the plant roots or open-loop system

roots required in the proposed criterions can be received from the experiment.

The number of the roots of the plant or the open-loop system with the positive real plant which is required for the Nyquist criterion cannot be received from experiment.

## APPENDIX A. THE PROOF OF THE THEOREM 4.1.

Multiply (31) by j and add (30). Then we derive the equation system (24), (29) which is equal to (23),(29). Multiply the each equation of system (23) by the corresponding polynomial $d(s_k)$ where $s_k=\lambda+\omega_k$ $(k=\overline{1,n})$. (In accordance with the remark 2.1 $d(s_k)\neq 0$ $(k=\overline{1,n})$). Then we obtain the equation system

$$a(s_k)\delta_2(s_k)-k(s_k)\delta_1(s_k)=0 \ (k=\overline{1,p}), \quad \delta_{1,2n-1}=1 \qquad (A.1)$$

The identity (20) can by expressed in the following form

$$\sum_{i=0}^{2p-1} x_\alpha s^\alpha=0 \qquad (A.2)$$

where

$$x_\alpha=\sum_{i=0}^{\gamma}(a_{\alpha-i}\delta_{2,i}-k_i\delta_{1,\alpha-i}) \ (\alpha=\overline{0,2p-1}). \qquad (A.3)$$

$(a_{\alpha-i}=\delta_{1,\alpha-i}=0$, if $\alpha-i<0$ or $\alpha-i>2n-1)$.

The coefficients of the polynomials (21) are the solution of the of the equation system

$$x_\alpha=0 \ (\alpha=\overline{0,2p-1}), \quad \delta_{1,2n-1}=1 \qquad (A.4)$$

The equations (A.1) is represented similarly (A.2)

$$\sum_{i=0}^{2p-1} x_\alpha s_k^\alpha=0 \quad (k=\overline{1,p}), \quad \delta_{1,2n-1}=1 \qquad (A.5)$$

It is sufficient to prove now that (A.4) is the unique solution of (A.5).

Restricting to the case $\lambda=0$ for simplisity, we rewrite (A.5) as

$$\sum_{i=0}^{p-1}\omega_k^{2i}(-1)^i x_{2i}=0, \quad \sum_{i=0}^{p-1}\omega_k^{2i+1}(-1)^i x_{2i+1}=0,$$
$$\delta_{1,2n-1}=1, \quad (k=\overline{1,p}) \qquad (A.6)$$

Rewrite (A.6) in the form

$$Mx=0, \quad \delta_{1,2n-1}=1 \qquad (A.7)$$

where

$$M=\begin{bmatrix} 1 & 0 & -\omega_1^2 & 0 & \omega_1^4 \ldots 0 & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \omega_p^2 & 0 & \omega_p^4 \ldots 0 & \\ 0 & \omega_1 & 0 & -\omega_1^3 & 0 & \ldots\omega_1^{2p-1}(-1)^{n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \omega_p & 0 & -\omega_p^2 & 0 & \ldots\omega_p^{2p-1}(-1)^{n-1} \end{bmatrix}; \quad x=\begin{bmatrix} x_0 \\ \vdots \\ x_{p-1} \\ x_p \\ \vdots \\ x_{2p-1} \end{bmatrix} \qquad (A.8)$$

The determinant of matrix M can be transformed as

$$\det M=\det\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}=\det M_1 \det M_2 \qquad (A.9)$$

where $\det M_1$ and $\det M_2$ are the Vandermonde determinants.

If $\omega_i\neq\omega_k$ $(i\neq k)$ then $\det M_i\neq 0$ $(i=\overline{1,p})$ and consequently $\det M\neq 0$.

This means that (A.4) is the unique solution (A.5). The proof for $\lambda\neq 0$ is similarly.

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