FREQUENCY ADAPTIVE CONTROL.

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Abstract. The plants with the uncertain coefficients in the presence of the bounded disturbance are considered. The direct method of the frequency adaptive control is developed for the nonminimum-phased plants. The frequency adaptive control bases oneself upon the frequency domain parameters which are the signals of the Fourier filter outputs. The experimental determination of these parameters is possible if the plant unstability degree estimation is known. The approach to this estimation determination is given and its convergency is proved.

Keywords. Adaptive control, external disturbances, frequency responses, nonminimum-phased systems, modal control.

1. INTRODUCTION

The adaptive control theory evalution may be divided on two stages.

At the first stage, so-called "ideal case" (Narendra and Annaswamy, 1986) which supposes that external disturbances are either absent (Anderson and others, 1986) or "white noise" (Iserman, 1981) was investigated. This stage was finished in 1980.

At the second stage the more real case is considered. In this case external disturbances are any bounded functions.

So, authors (Narendra and Annaswamy, 1986) have deduced the disturbance maximal amplitude for which the processes in the model reference adaptive system are bounded. Other papers of this direction has been refered by Ortega and Tang (1989).

The new method based on the recurrent targetal inequalities was supposed by Fomin, Fradkov and Jakubowich (1981), Jakubowich (1988).

The different approach originated from the notion the frequency domain parameters was given by Alexandrov (1989, 1991a). These parameters are easy determined with the experiment as the Fourier filter outputs. It was shown that the filter signals convergate to these parameters for the wide class of bound disturbation. Using this fact the direct method of the frequency adaptive control for the minimum-phase plants was derived by Alexandrov (1991a).

The aim of this paper is the development of this approach for the nonminimum-phased plants.

2. PROBLEM STATEMENT

The Model of the Control Plant

Consider the completely controllable plant described by the equation

$$y^{(n)}+d_{n-1}y^{(n-1)}+\ \dots\ +d_1\dot{y}+d_0y =$$

$$= k_\gamma u^{(\gamma)}+\ \dots\ k_0u + m_\alpha f^{(\alpha)}+\ \dots\ + m_0 f \qquad (1)$$

in which y(t) is the measured output, u(t) is the controlled input, f(t) is the external disturbance.

The coefficients d_i and k_j ($i=\overline{0,n-1}$, $j=\overline{0,\gamma}$) of the differential equation (1) are unknown.

The bounds of the coefficient m_0 and the least root estimation $\bar{s}^*$ of polynomial $m(s)=m_\alpha s^\alpha+\dots+m_0$ with the uncertain parameters are given

$$|m_0| \le m_0^*, \qquad (2)$$

$$|\bar{s}| \le \bar{s}^*,\quad \bar{s}=\min\{|\bar{s}_1|,\dots,|\bar{s}_\alpha|\}, \qquad (3)$$

where $\bar{s}_i(i=\overline{1,\alpha})$ are the roots of polynomials $m(s)$, m_0^* and s^* are the given numbers.

The external disturbance is any bounded function

$$|f(t)| \le f^*, \qquad (4)$$

(where f^* is the specified number) which may be represented as

$$f(t)=\sum_{i=1}^{\gamma 1}\delta_i^s \sin\,\omega_i^f t + \sum_{j=1}^{\gamma 2}\delta_j^c\cos\,\omega_j^f t, \qquad (5)$$

where numbers δ_i^s, δ_j^c ($i=\overline{1,\gamma_1}$, $j=\overline{1,\gamma_2}$), and $\omega_i^f(i=\overline{1,\max(\gamma_1,\gamma_2)})$ are unknown, however

$$\sum_{i=1}^{\gamma 1}|\delta_i^s| + \sum_{j=1}^{\gamma 2}|\delta_j^c| \le f^*. \qquad (6)$$

The Control Problems

We shall search for an equation of a controller in the form

$$g_{n-1}u^{(n-1)}+\ \dots\ +g_0u=r_{n-1}y^{(n-1)}+\ \dots\ +r_0y, \qquad (7)$$

in which g_i and r_j ($i, j=\overline{0,n-1}$) are the searched coefficients.

Problem 2.1 (Problem of the accurate adaptive control).

Find the controller coefficients such that the system (1) and (7) satisfies the requirement to accuracy

$$|y(t)| \le y^*, \qquad t \ge \bar{t}, \qquad (8)$$

where y^* is the specified number, $\bar{t}$ is some time moment. ▲

3. THE CONTROL FOR THE KNOWN PLANT COEFFICIENT

Modal Control

If the coefficients d_i and k_j ($i=\overline{0,n-1}$, $j=\overline{0,\gamma}$) are known then the algorithm of the modal control consists of the following steps:

S t e p 1. Compose the identity

$$d(s)g(s)-k(s)r(s)=\delta(s). \qquad (9)$$

S t e p 2. Check the coefficients by the equal degrees of symbol s in the right and left parts of this identity and form of the next system of linear algebraic equations

$$\sum_{i=0}^{n} d_i g_{\alpha-i} - \sum_{l=0}^{\gamma} k_l r_{\alpha-l} = \delta_\alpha, \quad (\alpha=\overline{0,2n-1}) \qquad (10)$$

S t e p 3. Solve system (10). As the plant (1) is completely controllable and the degree of polynomial $r(s)$ less than the degree of polynomial $d(s)$ then the solution of system (10) exists, and it is unique (Jakubowich, 1988; Wolovich, 1974).

The Accurate Control: Minimum-Phased Plant

We shall find the coefficients of the controller (7) for the minimum-phased plant (in which $k(s)$ is Hurwitz's polynomial) using the identity

$$d(s)g(s)-k(s)r(s)=\varkappa(s)k(s)\tilde{\psi}(s) \qquad (11)$$

where

$$\tilde{\psi}(s)=s^n+\tilde{\psi}_{n-1}s^{n-1}+\ \dots\ +\tilde{\psi}_0=\tilde{\psi}_0\prod_{i=1}^{n}(T_is+1),$$

$$\prod_{i=1}^{n}T_i\tilde{\psi}_0=1; \quad T_i>0 \qquad (i=1,n); \qquad (12)$$

$$\varkappa(s)=(\tau s+1)^{n-\gamma-1}, \qquad \tau>0. \qquad (13)$$

Assertion 3.1 If the coefficients of the polynomial $\tilde{\psi}(s)$ satisfy the next conditions

$$\tilde{\psi}_0 \ge \frac{m_0^* f^*}{y^*}, \qquad (14)$$

$$T_i \ge \frac{1}{\bar{s}^*} \quad (i=\overline{1,\alpha}), \qquad (15)$$

$$\prod_{i=\alpha+1}^{n} T_i=\frac{1}{\tilde{\psi}_0\prod_{i=1}^{\alpha}T_i}, \qquad (16)$$

τ is sufficiently small number, then the system (1), (7) with the controller which is found from the identity (11) satisfies the requirement (8) to accuracy. ▲

Proof. The measured output $y(s)$ connects with the external disturbance as

$$y(s)=\frac{m(s)g(s)}{\varkappa(s)k(s)\tilde{\psi}(s)}f(s). \qquad (17)$$

Let $\tau=0$. Then the identity (11) has the obvious solution

$$g(s)=k(s), \ r(s)=d(s)-\tilde{\psi}(s). \qquad (18)$$

We substitute these expressions in (17) and obtain

$$y(s) = \frac{m(s)}{\tilde{\psi}(s)}f(s) =$$

$$=\frac{m_0}{\tilde{\psi}_0}\frac{\prod_{i=1}^{\alpha_1}(\check{T}_is+1)\prod_{i=1}^{\alpha_2}(\check{\check{T}}_i^2s^2+2\check{\check{T}}_i\check{\check{\xi}}_is+1)}{\prod_{i=1}^{n}(T_is+1)}f(s), \quad \alpha_1+2\alpha_2=\alpha. \qquad (19)$$

The unequality (3) means that

$$\frac{1}{\bar{s}^*}\ge \max\{\check{T}_1,\dots,\check{T}_{\alpha 1},\check{\check{T}}_1,\dots,\check{\check{T}}_{\alpha 2}\} \qquad (20)$$

and therefore the condition (17) gives

$$\max_{0\le\omega^f\le\infty}\left|\frac{\prod_{i=1}^{\alpha 1}(\check{T}_ij\omega^f+1)\prod_{j=1}^{\alpha 2}(\check{\check{T}}_i^2(j\omega^f)^2+2\check{\check{T}}_i\check{\check{\xi}}_j(j\omega^f)+1}{\prod_{i=1}^{\alpha}(T_ij\omega^f+1)}\right|\le 1 \qquad (21)$$

If the external disturbance has the form (6) then taking account (19) and (21) we derive an expression of the output for the steady state

$$|y(t)|\le\frac{|m_0|}{\tilde{\psi}_0}\left[\sum_{i=1}^{\gamma 1}|\delta_i^s|+\sum_{j=1}^{\gamma 2}|\delta_j^c|\right]\le\frac{m_0^*}{\tilde{\psi}_0}f^*\le y^*. \qquad (22)$$

For sufficiently small $\tau\neq 0$ the *solution of the* identity (11) has the form

$$g(s)=(\varkappa(s)+o^{\varkappa}(s))k(s), \ r(s)=d(s)-\tilde{\psi}(s)+o^{r}(s) \qquad (23)$$

where $o^{\varkappa}(s)$ and $o^{r}(s)$ are the polynomials of the degrees $n-\gamma-1$ and $n-1$ respectively.

Substituting (23) in (17) we obtain a expression which is near to (22) since the coefficients of the polynomials (23) are depend on τ continuously (Alexandrov, 1989). ▲

PID-Controller

Let $f(t)$ is step disturbance: $f(t)=f^*$ for $t\ge t_0$ and

$f(t)=0$ for $t<t_0$. The requirement (8) to accuracy is fulfilled if

a) the coefficient $\tilde{\psi}_0$ in the polynomial (12) satisfies unequality (14),

b) T_i $(i=\overline{1,n})$ are any positive numbers,

c) τ is a sufficiently small number.

However, other way of reaching of the objective (8) for the step disturbance exists. The basis of this way is PID-Controller.

To build such controller we find modal control for the next "widened plant"

$$d(s)y=k(s)u+m(s)f, \quad \dot{\bar{y}}=y, \tag{24}$$

which may by rewritten as

$$\bar{d}(s)\bar{y}=k(s)u+m(s)f, \tag{25}$$

where

$$\bar{d}(s)=d(s)s. \tag{26}$$

If $k_0\neq 0$ then "plant" (25) is the completely controllable. The identity (9) has now the form

$$\bar{d}(s)\bar{g}(s)-k(s)\bar{r}(s)=\bar{\delta}(s), \tag{27}$$

where $\bar{\delta}(s)$ is Hurwitz polynomial of degree 2n.

Solving this identity we derive the coefficients of an equation $\bar{g}(s)u=\bar{r}(s)\bar{y}$. Then the searched PID-Controller is described as

$$s\bar{g}(s)u=\bar{r}(s)y. \tag{28}$$

The output of plant with this controller represents in the view

$$y(s) = \frac{sg(s)m(s)}{\bar{\delta}(s)} f(s) \tag{29}$$

from which follows that $\lim_{t\to\infty} y(t) = 0$.

4. FREQUENCY DOMAIN PARAMETERS

Notions

Definition 4.1 If the plant (1) is unstable then a number

$$s^* = \max \{\mathrm{Re}\, s_1, \ldots, \mathrm{Re}\, s_n\}, \tag{30}$$

where s_i $(i=\overline{1,n})$ are the roots of the polynomial d(s) are called the unstability degree of plant (1). ▲

Let $c_0>0$ denotes the estimation of an upper boundary of the unstability degree:

$$c_0 \geq s^*. \tag{31}$$

Definition 4.2 A set of the 2n numbers

$$\alpha_k=\mathrm{Re}\,w(\lambda+j\omega_k), \; \beta_k=\mathrm{Im}\,w(\lambda+j\omega_k), \; (k=\overline{1,n}), \; \lambda\geq c_0 \tag{32}$$

are called the frequency domain parameters.

Here $w(s)=k(s)/d(s)$ is the transfer function of plant (1), ω_k $(k=\overline{1,n})$ are some positive numbers which are below called the test frequences. ▲

The estimation c_0 is supposed to be known in this definition. Such assumption may be cancelled as it is shown in section 6.

The Experimental Determination

The method of the experimental determination of the frequency domain parameters consists in the following.

Apply to the input of plant (1) the test signal

$$u=e^{\lambda t}\sum_{k=1}^{n}\rho_k\sin(\omega_k+\varphi_k), \quad \lambda>c_0, \tag{33}$$

where ρ_k, ω_k and φ_k $(k=\overline{1,n})$ are given numbers.

The signal y(t) of the plant output after multiplication of it by $e^{-\lambda t}$ is applied to the input of the Fourier filter (Eykhoff, 1974). Its outputs give the estimations of the frequency domain parameters

$$\hat{\alpha}_k(t_\phi)=\frac{2}{\rho_k(t_\phi-t_0)}\int_{t_0}^{t_0+t_\phi} y(t)e^{-\lambda t}\sin(\omega_k t+\phi_k)dt \; (k=\overline{1,n}), \tag{34}$$

$$\hat{\beta}_k(t_\phi)=\frac{2}{\rho_k(t_\phi-t_0)}\int_{t_0}^{t_0+t_\phi} y(t)e^{-\lambda t}\cos(\omega_k t+\phi_k)dt \; (k=\overline{1,n}). \tag{35}$$

Theorem 4.1. (Alexandrov, 1991a) The estimations of the frequency domain parameters of plant (1) in the presents of disturbance (5) have the next property

$$\lim \hat{\alpha}_k(t_\phi)=\alpha_k, \quad \lim \hat{\beta}_k(t_\phi)=\beta_k, \quad k=(\overline{1,n}) \quad ▲ \tag{36}$$

This property is well known (Eykhoff, 1974) for a stable plant. In fact, if we apply to the plant (1) the harmonic signal $u=1\cdot\sin\omega_k t$ then its output has the form

$$y(t)=\alpha_k\sin\omega_k t+\beta_k\cos\omega_k t+x(t), \tag{37}$$

where $x(t)$ is vanish function ($\lim_{t\to\infty} x(t)=0$). Applying this signal to the Fourier filter we obtain a numbers α_k and β_k if the time **t** is sufficiently large. If $f(t)\neq 0$ then the described experiment may give the shift of the estimations. In case when in (5) $\omega_i^f\neq\omega_k$ $(i=\overline{1,\max(\gamma_1,\gamma_2)}, k=\overline{1,n})$ this shift equals zero. However, this unequality can not be verified as ω_i^f $(i=\overline{1,\max(\gamma_1,\gamma_2)})$ are unknown.

A few of the methods may be proposed to obtain the unshifted estimates of frequency domain parameters of the stable plants. So, the test signal (33) for $\lambda>0$ may be used for such plants. Other two methods have been described by Alexandrov (1991a).

5. FREQUENCY ADAPTIVE CONTROL

The Accurate Control: Minimum-Phased Plant

Solve the problem 2.1 for the minimum-phased plant which is specified by the frequency domain parameters α_k and β_k $(k=\overline{1,n})$.

For this objective we divide the identity (11) by the polynomial d(s) and derive the expression

$$g(s)-w(s)r(s)=w(s)x(s)\tilde{\psi}(s), \quad (38)$$

which for $s=\lambda+j\omega_k$ $(k=\overline{1,n})$ has the form

$$\sum_{i=0}^{n-1} g_i[\lambda+j\omega_k]^i - (\alpha_k+j\beta_k)\sum_{i=0}^{n-1} r_i[\lambda+j\omega_k]^i = \quad (39)$$

$$= (\alpha_k+j\beta_k)\sum_{i=0}^{2n-\gamma-1}\psi_i[\lambda+j\omega_k]^i \quad (k=\overline{1,n}),$$

where ψ_i $(i=\overline{1,2n-\gamma-1})$ are the coefficients of the polynomial $x(s)\tilde{\psi}(s)$.

The system (39) contains the 2n linear algebraic equations for calculating the real coefficients of controller (7). It may be rewritten in the form of two subsystems

$$\sum_{i=0}^{n-1}\rho_i(\omega_k)\, g_i - \sum_{i=0}^{n-1}[\alpha_k\rho_i(\omega_k)-\beta_k\mu_i(\omega_k)]\, r_i = \quad (40)$$

$$= \sum_{i=0}^{2n-\gamma-1}[\alpha_k\rho_i(\omega_k)-\beta_k\mu_i(\omega_k)]\,\psi_i, \quad (k=\overline{1,n})$$

$$\sum_{i=0}^{n-1}\mu_i(\omega_k)\, g_i - \sum_{i=0}^{n-1}[\alpha_k\mu_i(\omega_k)+\beta_k\rho_i(\omega_k)]\, r_i = \quad (41)$$

$$= \sum_{i=0}^{2n-\gamma-1}[\alpha_k\mu_i(\omega_k)+\beta_k\rho_i(\omega_k)]\,\psi_i, \quad (k=\overline{1,n})$$

where $\rho_i(\omega_k)=\mathrm{Re}[\lambda+j\omega_k]^i$, $\mu_i(\omega_k)=\mathrm{Im}[\lambda+j\omega_k]^i$ $(i=\overline{1,n})$.

Theorem 5.1 (Alexandrov, 1989) If the plant (1) is completely controllable then system (40), (41) has the solution which is unique and it coincide with the solution of system (10) for arbitrary ω_k $(k=\overline{1,n})$ and $\lambda>c_0$. ▲

And so, the direct algorithm of the frequency adaptive accurate control of the minimum-phased plant consists from the next steps

S t e p 1. Find the estimations of the frequency domain parameters $\hat{\alpha}_k(t_\Phi)$ and $\hat{\beta}_k(t_\Phi)$ $(k=\overline{1,n})$.

The time t_Φ of the experiment may be determined from the inequalities

$$|\hat{\alpha}_k(t_\Phi)-\hat{\alpha}_k(t_\Phi-\Delta t|\le\varepsilon, \quad |\hat{\beta}_k(t_\Phi)-\hat{\beta}_k(t_\Phi-\Delta t|\le\varepsilon \quad (k=\overline{1,n}) \quad (42)$$

in which ε is sufficiently small number, Δt is some number.

S t e p 2. Solve the system (40), (41). The numbers α_k and β_k $(k=\overline{1,n})$ are substituted by theirs estimations.

S t e p 3. Close the plant (1) using the controller (7) with the coefficients which were derived on the step 2.

The time $\bar{t}$ of the attainment of the control target (8) consists from the intervals:

$$\bar{t} = t_\Phi + t_s + t_r, \quad (43)$$

where t_s is a time of the system (41), (42) solution, t_r is the response time of the system (1), (7).

PID-Controller for Nonminimum-Phased Stable Plant

Consider the case when the plant (1) is asymptotical stable (d(s) is Hurwitz polynomial) but it is nonminimum-phased (k(s) is not Hurwitz polynomial). Let too the external disturbance is the piece-constant function with the intervals of constancy more then $\bar{t}$.

PID-Controller which assure the accurate control is built by the following way.

Let in the identity (27)

$$\bar{\delta}(s)=d(s)\bar{\psi}(s) \quad (44)$$

where $\bar{\psi}(s)$ is some Hurwitz polynomial of the degree n. Dividing this identity by sd(s) we obtain for $s=\lambda+j\omega_k$ $(k=\overline{1,n})$ the system of equations

$$\sum_{i=0}^{n}\bar{g}_i[\lambda+j\omega_k]^i - (\bar{\alpha}_k+j\bar{\beta}_k)\sum_{i=0}^{n}\bar{r}_i[\lambda+j\omega_k]^i = \quad (45)$$

$$= \frac{1}{\lambda+j\omega_k}\sum_{i=0}^{n}\bar{\varphi}_i[\lambda+j\omega_k]^i \quad (k=\overline{1,n+1}),$$

in which numbers $\bar{\alpha}_k$ and $\bar{\beta}_k$ $(k=\overline{1,n+1})$ are calculated with employment the estimates of the frequency domain parameters as

$$\bar{\alpha}_k = \frac{\alpha_k\lambda+\beta_k\omega_k}{\lambda^2+\omega_k^2}, \quad \bar{\beta}_k = \frac{-\alpha_k\omega_k+\lambda\beta_k}{\lambda^2+\omega_k^2} \quad (k=\overline{1,n+1}) \quad (46)$$

These formulae follow from the expression $\bar{w}(s)=w(s)/s$.

6. EXPERIMENTAL DETERMINATION OF C_0

The estimation of the unstability degree is usually unknown but it may be determined as the result of the experiments.

The idea of such experiments has been described by Alexandrov (1991b). It consists in following. The test signal $u(t)=e^{\lambda_1 t}\sin\omega t$ (where λ_1 is some positive number) is applied to the plant (1). If the function $\tilde{y}^{(1)}(t)=y(t)e^{-\lambda_1 t}$ increases then we have to apply the signal $u(t)=e^{\lambda_2 t}\sin\omega t$ (where $\lambda_2 > \lambda_1$) *and to repeat the experiments till a* number λ_N, for which the product $\tilde{y}^{(N)}(t)=y(t)e^{-\lambda_N t}$ is bounded function, will be obtain. The number λ_N is adopted as c_0.

Such method find difficulty in doing the implementation because the time of every from this experiments is restrictly. It is connected with the natural bounds for y(t) and u(t):

$$|y(t)| < q_y, \quad |u(t)| < q_u, \quad (47)$$

where q_y and q_u are the given numbers which characterize the saturation of the measuring and drive devices.

Therefore every experiment is ended in a moment $t_e^{(i)}$ $(i=\overline{1,N})$ when is fulfilled one from the equalities

$|y(t_e^{(i)})|=q_y$ or $|u(t_e^{(i)})|=q_u$ $(i=\overline{1,n})$ (48)

To increase the test times we will apply the signal $u=\rho\sin\omega t$ (where ρ is constant) to plant (1) and estimate of the functions $\tilde{y}^{(i)}(t)=y(t)e^{-\lambda_i t}$ $(i=\overline{1,N})$ rising λ as early.

Theorem 6.1. It exists the value λ such that the plant output which is excited by the test signal $u(t)=\rho\sin\omega t$ satisfies the condition

$$|y(t)e^{-\lambda t}| \le \delta^*, \quad t > t^*, \tag{49}$$

where δ^* and t^* are the arbitrary numbers (for example: $\delta^* = y^*$, $t^* < t_e^{(i)}$, $i=\overline{1,N}$). ▲

The proof is given by the appendices.

7. EXAMPLES

Example 7.1.

Consider the plant (Fomin, Fradkov and Jakubowich, 1981)

$$\ddot{y} + d_0 y = k_1\dot{u} + k_0 u + f. \tag{50}$$

Let the following assumptions are valid:

a) it is minimum-phased plant,

b) its instability degree estimation $C_0=6$,

c) the external disturbance satisfy the restriction $|f(t)| \le 10$.

Find the controller

$$g_1\dot{u} + g_0 u = r_1\dot{y} + r_0 y \tag{51}$$

such that

$$|y(t)| \le 0.2, \quad t > \bar{t}. \tag{52}$$

The numerical experiments were performed. The coefficients of the plant (50) were $d_0=-16$, $k_0=30$, $k_1=5$ and external disturbances $f(t)=10\sin 9t$. The experimentor was not informed about these data.

These experiments with the test frequency $\omega_1=3s^{-1}$ and $\omega_2=6s^{-1}$ under the condition $\lambda=6$ have given the next estimations of the frequency domain parameters $\hat{\alpha}_1=0.855$, $\hat{\beta}_1=-1.4$, $\hat{\alpha}_2=0.285$, $\hat{\beta}_2=-0.851$.

Using (14) we have found $\tilde{\psi}_0=50$ and adopted $\tilde{\psi}_1=14$. Then the equations (40), (41) were solved and the following coefficients of the controller (51) were derived $g_1=4.98$, $g_0=27.6$, $r_1=-14.4$, $r_0=-63$.

The simulation results of the system (50), (51) have shown that its steady-state error equals 0.0746. Therefore the requirement (52) to accuracy has been fulfilled.

Example 7.2.

The plant to be considered is described by the following transfer function (Saad, 1991)

$$w(s) = k\,\frac{(Ts-1)\omega_0^2}{(Ts+1)(s^2+2\xi\omega_0 s+\omega_0^2)} \tag{53}$$

This plant is asymptotical stable ($T>0$, $\xi>0$, $\omega_0>0$) but it is nonminimum-phased.

Find the PID-Controller

$$(\bar{g}_3 s^3+\bar{g}_2 s^2+\bar{g}_1 s+\bar{g}_0)su=(\bar{r}_3 s^3+\bar{r}_2 s^2+\bar{r}_1 s+\bar{r}_0)(y+r^*) \tag{54}$$

(in which r^* is the step set-point sequence) such that, beginning with a moment $\bar{t}$, the tracking error becomes equal zero. This requirement has to be fulfilled for the step external disturbances which satisfies the condition $|f(t)| \le 5$.

As in the example 7.1 were performed the numerical experiments for $T=1$, $\omega_0=15$, $k=3$, $\xi=1$.

The experiments with the test frequency $\omega_1=1$, $\omega_2=5$, $\omega_3=10$, $\omega_4=20$ have given the estimations $\hat{\alpha}_1=0.4$, $\hat{\beta}_1=2.96$, $\hat{\alpha}_2=2.66$, $\hat{\beta}_2=-0.65$, $\hat{\alpha}_3=1.26$, $\hat{\beta}_3=-1.69$, $\hat{\alpha}_4=-0.196$, $\hat{\beta}_4=-1.06$.

Then the frequency domain parameters α_k and β_k $(k=\overline{1,n})$ were calculated on the formulas (46). Adopting $\tilde{\psi}(s)=(0,1s+1)^4$ we have solved the equations (45).

The next coefficients of controller (54) were obtained $\bar{g}_0=1.47$, $\bar{g}_1=0.0062$, $\bar{g}_2=0.0056$, $\bar{g}_3=0.58\cdot10^{-4}$, $\bar{r}_0=0.34$, $\bar{r}_1=0.36$, $\bar{r}_2=0.027$, $\bar{r}_3=-0.485\cdot10^{-3}$.

The simulation of the system (53), (54) has shown that the process in this system attains the wished value $y=r^*=10$ in time 4T.

Remark. The examples 7.1 and 7.2 have been solved with CHAR (special PC software).

8. CONCLUSION

The way of the experimental determination of the plant unstability degree estimation has been given. Convergency of this way has been proved.

Direct algorithm of the frequency adaptive control to be proposed by Alexandrov (1991a) for the minimum-phased plant has been now developed for nonminimum-phased plant. The adaptive PID-Controller algorithm for the stable plant has been derived.

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APPENDICES

The Proof of the Theorem 6.1

Convert the equation (1) to the next form

$$\dot{x} = Px + bu + \psi f, \qquad y=l^T x, \tag{A.1}$$

where x(t) is the n-dimensional vector, b, ψ and l are the n-dimensional vectors and P is matrix n×n.

Let, for simplicity, that the external disturbance (5) is single-frequency: $f(t)=\delta\sin\omega^f t$, $|\delta|\le f^*$.

Then the plan output is excited by this disturbance and the test signal $u(t)=\rho\sin\omega t$. It has the form (Alexandrov, 1989):

$$y(t)=l'x(0)e^{Pt}-l'e^{Pt}[\mathrm{Im}\beta(j\omega)+\mathrm{Im}\beta_f(j\omega^f)]+\mu(t) \tag{A.2}$$

in which

$$\mu(t)=\rho[\mathrm{Re}w(j\omega)\sin\omega t+\mathrm{Im}w(j\omega)\cos\omega t]+ \\ +\delta[\mathrm{Re}w_f(j\omega^f)\sin\omega^f t+\mathrm{Im}\, w_f(j\omega^f)\cos\omega^f t] \tag{A.3}$$

where

$$\beta(s)=(Es-P)^{-1}b, \qquad \beta_f(s)=(Es-P)^{-1}\psi, \tag{A.4}$$

$$w(s)=l'\beta(s), \qquad w_f(s)=l'\beta_f(s)=\frac{m(s)}{d(s)}. \tag{A.5}$$

The expression (A.2) may be rewritten in more detailed view as

$$y(t) = \sum_{q=1}^{n} c_q^s\sin\omega_q^0 + c_q^c\cos\omega_q^0)e^{a_q^0 t} + \mu(t), \tag{A.6}$$

where the numbers c_q^s and c_q^c $(q=\overline{1,n})$ depended on the vectors x(0), $\mathrm{Im}\beta_i(j\omega)$ and $\mathrm{Im}\beta_f(j\omega^f)$; $a_q^0+j\omega_q^0=s_q$ $(q=\overline{1,n})$ are the roots of the polynomial det(Es-P). The roots is supposed to be simple.

Consider the function now

$$\tilde{y}(t)=y(t)e^{-\lambda t}=\sum_{q=1}^{n}(c_q^s\sin\omega_q^0+c_q^0\cos\,\omega_q^0)e^{(a_q^0-\lambda)t}+e^{-\lambda t}\mu(t) \tag{A.7}$$

Taking info account inequality

$$|\mu(t)| \le \rho|w(j\omega)| + f^*|w_f(j\omega^f)| \tag{A.8}$$

we obtain

$$|\tilde{y}(t)| \le \sum_{q=1}^{n} e^{(a_q^0-\lambda)t}\{|c_q^s| + |c_q^c|\} + \\ + e^{-\lambda t}\{\rho|w(j\omega)| + f^*|w_f(j\omega^f)|. \tag{A.9}$$

It is obvious that for the arbitrary values δ^* and t^* exists the number λ which assures the fulfilment of unequality

$$\sum_{q=1}^{n} e^{(a_q^0-\lambda)t^*}\{|c_q^s| + |c_q^c|\} + e^{-\lambda t}\{\rho|w(j\omega)| + \\ + f^*|w_f(j\omega^f)|\} \le \delta^* \tag{A.10}$$

The proof for the repeated roots of the polynomial det(Es-P) is similarly.