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DYNAMIC SYSTEM IDENTIFIER. EXPERIMENTAL INVESTIGATION

Results of experimental investigations of the dynamic system identifier are given. The identifier is a C++ program based on finite-frequency identification algorithm. An external disturbance, which is an unknown-but-bounded function, is fed to the plant. The experimental investigations show a high accuracy of identification.

1. INTRODUCTION

Real plant identification as rule pass on conditions of unknown-but-bounded external disturbances (with unknown stochastic characteristics) and bounded duration of identification. In this case it is applied an active identification with using a testing signal.

One of way of the active identification is a finite-frequency identification [1, 2], where testing signal is several sequences of harmonics. The number of harmonics does not exceed a plant order. Their amplitudes and frequencies are automatically tuned [3] in the process of identification.

IDP-2 C++ program based on above method has been developed. This program installs on PC (it may be a single-board PC104 or another similar or even a DSP processor with C++ support). PC should have analog-digital and digital-analog converters (ADC and DAC) for the connection with real plant. It allows identify wide class of control plant with different physical nature.

In papers a result of experimental investigations of identifies using IDP-2 program are given. In our earlier work [4] other algorithm of finite-frequency identification without any self-tunings has been realized.

2. PURPOSE AND FIELD OF APPLICATION

2.1. PURPOSE

A discrete plant is described by the following difference equation:

$$\begin{aligned} &y[kh] + d_1 y[k(h-1)] + \ldots + d_n y[k(h-n)] = k_1 u[k(h-1)] + \ldots \\ &\ldots + k_n u[k(h-n)] + f[k(h-1)] \quad (k = 0,1,2,\ldots), \end{aligned} \tag{1}$$

where $y[kh]$ is the output measured in samples kh (where h is a sampling interval), $u[kh]$ is a test-

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ing signal, $f[kh]$ is an unknown-but-bounded disturbance:

$$|f[kh]| \le f^* \quad (k = 0,1,2,\ldots),$$

where f^* is given, coefficients d_i and k_i $(i = \overline{1,n})$ are unknown numbers, n is the known order of plant.

Input and output are bounded:

$$|u[kh]| \le u\_, \ |y[kh]| \le y\_ \ (k = 0,1,2,\ldots), \quad (2)$$

where $u\_$ and $y\_$ are known input and output range limits.

2.2. SOURCES (SOURCE DATA, INPUT DATA)

The sources for DPI2D are:

- plant order n;
- sampling interval h;
- bounds of signals (2).

Along these, the next parameters must be given (they are being determined by means of system modeling in ADAPLAB-MD software, the meaning of these parameters is given below):

- a number of the base periods p_{init};
- a value ε (is given below);
- bounds of the dynamic correlation coefficients δ_α and δ_β.

3. A PROGRAM STRUCTURE AND ALGORITHM

An identification algorithm DPI2D is based on a finite-frequency identification method with self-tuning of test signal amplitudes and durations [3]. A block diagram of DPI2D appears on figure 1.

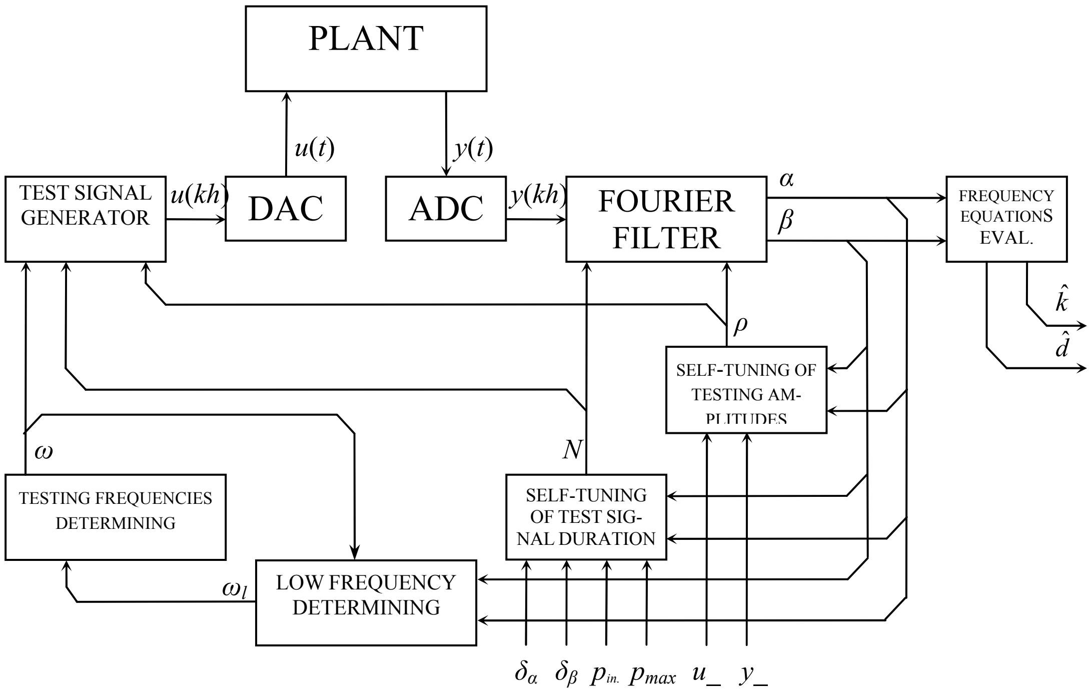

Fig. 1. DPI2D block diagram

3.1. TEST SIGNAL GENERATOR

A test signal generator forms the single harmonic:

$$u(kh) = \rho \sin \omega kh, \quad k = \overline{0, N-1}, \tag{3}$$

where ρ is a current test amplitude (it is determined by self-tuning of testing amplitudes block), ω is a current testing frequency, N is a number of sampling intervals in duration (it's determined by self-tuning of test signal duration block and determines the duration).

3.2. FOURIER FILTER

A Fourier filter evaluates a frequency parameters of the plant (1) approximately:

$$\begin{aligned} \hat{\alpha}(N) &= \frac{2}{\rho N} \sum_{k=0}^{N-1} y(kh) \sin \omega kh \\ \hat{\beta}(N) &= \frac{2}{\rho N} \sum_{k=0}^{N-1} y(kh) \cos \omega kh \end{aligned} . \tag{4}$$

3.3. A FREQUENCY EQUATION SOLVER

A solver estimates plant coefficients (1), using plant frequency parameters (4) on the input. A discrete-time transfer function is:

$$W(z^{-1}) = \frac{k_1 z^{-1} + \ldots + k_n z^{-n}}{1 + d_1 z^{-1} + \ldots + d_n z^{-n}} = \frac{k(z^{-1})}{1 + d(z^{-1})}, \tag{5}$$

where z is a multiplier, representing the time biasing forward on one sampling interval.

The frequency parameters (4) of plant (1) relates to its transfer function (5) so:

$$W(e^{-j\omega_i h}) = \frac{k(e^{-j\omega_i h})}{1 + d(e^{-j\omega_i h})} = \alpha_i + j\beta_i, \quad i = \overline{1,n}.$$

When the solver substitutes estimates (4) instead of α and β, it gets frequency equations, which solution is necessary for estimating parameters $\hat{k}$ and $\hat{d}$ of the transfer function (5):

$$\hat{k}(e^{-j\omega_i h}) - (\hat{\alpha}_i + j\hat{\beta}_i)\hat{d}(e^{-j\omega_i h}) = \hat{\alpha}_i + j\hat{\beta}_i, \quad i = \overline{1,n}. \tag{6}$$

In more detailed form it can be rewritten as:

$$\begin{cases} \sum_{\nu=1}^{n} \hat{k}_\nu \cos \nu\omega_i h - \sum_{\nu=1}^{n} \hat{d}_\nu \left(\hat{\alpha}_i \cos \nu\omega_i h + \hat{\beta}_i \sin \nu\omega_i h\right) = \hat{\alpha} \\ -\sum_{\nu=1}^{n} \hat{k}_\nu \sin \nu\omega_i h + \sum_{\nu=1}^{n} \hat{d}_\nu \left(\hat{\alpha}_i \sin \nu\omega_i h - \hat{\beta}_i \cos \nu\omega_i h\right) = \hat{\beta}_i \end{cases}, \quad i = \overline{1,n}. \tag{7}$$

3.4. A SELF-TUNING OF TESTING AMPLITUDES

This part is necessary for determine of the amplitude ρ. At first, plant (1) is being tested (3) with amplitude $\rho = u\_$:

$$u(kh) = u\_ \sin \omega kh, \quad k = \overline{0, N^{[0]} - 1},$$

where $N^{[0]}$ is an initial number of sampling intervals, which is determined by expression:

$$N^{[0]} = \frac{2\pi p_н}{\omega_l h},$$

where $p_н$ is an initial number of base periods T_l in the test signal:

$$T_l = \frac{2\pi}{\omega_l}.$$

where ω_l is a lower bound of a testing frequency (it is determined below).

Then verifying of the condition to the plant output occurs:

$$|y(kh)| \le y\_, \quad k = \overline{0, N^{[0]} - 1}. \tag{8}$$

If this condition is not true, the amplitude ρ should be decreased, until this condition became valid.

Finally, when desired amplitude ρ^* is found, an activity value of test signal is determined:

$$\chi = \frac{\left| y_{\max} - \overline{y}_{\max} \right|}{\left| y_{\max} \right|},$$

where

$$\overline{y}_{\max} = \max_{\frac{N^{[0]}}{2} \le k \le N^{[0]}} \left| \overline{y}(kh) \right|, \;\; y_{\max} = \max_{\frac{3N^{[0]}}{2} \le k \le 2N^{[0]}} \left| y(kh) \right|,$$

where $\overline{y}(kh)$ is so-called “native” plant output, when $u(kh) = 0$.

3.5. A SELF-TUNING TEST SIGNAL DURATION

Just after the amplitude ρ^* and the frequency ω of test signal (3) has been already found, duration of test signal is determined. For that, test signal (3) with amplitude $\rho = \rho^*$ and an initial number of samples N is applied to the plant:

$$u(kh) = \rho^* \sin \omega kh, \quad k = \overline{0, N^{[0]} - 1}.$$

Then the following conditions are checked:

$$K_\alpha(N) \le \delta_\alpha, \;\; K_\beta(N) \le \delta_\beta, \tag{9}$$

where $K_\alpha(N)$ and $K_\beta(N)$ is dynamic correlation coefficients:

$$K_\alpha(\tau) = \left| \frac{\overline{\hat{\alpha}}(N)}{\hat{\alpha}(N)} \right|, \;\; K_\beta(\tau) = \left| \frac{\overline{\hat{\beta}}(N)}{\hat{\beta}(N)} \right|,$$

where $\overline{\hat{\alpha}}(N)$ and $\overline{\hat{\beta}}(N)$ are Fourier filter outputs (4) when $y(kh) = \overline{y}(kh)$.

If at least one of conditions (9) is false, the number of the sampling interval redoubles and the test repeats while a desired number N^* will be achieved so that this both conditions are true.

3.6. A TESTING FREQUENCY DETERMINING

A testing frequency determining begins since determining of lower frequency ω_l.

An initial frequency $\omega^{[0]}$ is set for determining lower frequency ω_l, and following the algorithm in chapter 3.4, the amplitude of the test signal is determined. Then the test of the plant (1) is carried out and the frequency parameters (4) of the plant (1) are evaluated. Using these parameters the first approximation to lower frequency is evaluated by the expression:

$$\omega_l^{[0]} = \omega^{[0]} \frac{\hat{\alpha}^{[0]}}{\hat{\beta}^{[0]}}.$$

Then the frequency $\omega^{[0]}$ halves:

$$\omega^{[i+1]} = \omega^{[i]} / 2, \quad i = 0,1,\ldots$$

A new approximation with number *i* for lower frequency is determined again:

$$\omega_l^{[i]} = \omega^{[i]} \frac{\hat{\alpha}^{[i]}}{\hat{\beta}^{[i]}} .$$

Then following conditions are verified:

$$\frac{\omega_l^{[i-1]} - \omega_l^{[i]}}{\omega_l^{[i-1]}} \le \varepsilon . \qquad (10)$$

where ε is approximation degree to low frequency. While the condition (10) is false, number *i* incremented and test is repeated, until such a number *i* is found, so that condition (10) is true. At last, low frequency ω_l estimates is

$$\hat{\omega}_l = \omega_l^{[i]} .$$

Afterwards, other *n*-1 testing frequencies are determined by expression:

$$\lg \omega_k = \lg \hat{\omega}_l + (k-1)\frac{\lg \hat{\omega}_u - \lg \hat{\omega}_l}{n-1}, \quad k = \overline{1,n} , \qquad (11)$$

where $\hat{\omega}_u$ is an upper frequency, which is determined as

$$\hat{\omega}_u = M \hat{\omega}_l ,$$

where *M* is a known testing frequency band (high-to-low testing frequency ratio).

4. EXPERIMENTAL INVESTIGATIONS

4.1. A TEST BENCH

A test bench which is described in reference [5] has been used for an experimental investigation of DPI2D. It consists of personal computer (600 MHz P3 CPU) with an ADC-DAC board named L-780, which makes possible to use analog signals for connection to the plant, and, essentially, a physical plant model (PPM).

PPM [4] is an electrical device which consists of operational amplifiers and other passive components, so that its behavior corresponds to behavior of third-order dynamic system. PPM has a source of disturbance.

Continuous-time transfer function of PPM is:

$$W_{PPM}(s) = \frac{5000}{(s+2)(s^2+60s+2500)} . \qquad (12)$$

Its discrete-time transfer function is described as:

$$W_{PPM}(z)=\frac{0.00479z^2-0.01368z+0.002561}{z^3-1.726z^2+1.036z-0.2894}=\frac{0.0047902(z+2.655)(z+0.2013)}{(z-0.9608)(z^2-0.7647z+0.3012)}.$$

4.2. AN INITIAL DATA (SOURCES)

Following initial data, which is necessary to DPI2D operation selected as:

- sampling interval $h=0.002$;
- output range limit of the plant $y\_=1$;
- input range limit of the plant $u\_=4$;
- plant order $n=3$;
- testing frequency band $M=30$;
- number of the base periods $p_н=2$;
- approximation degree to low frequency $p_{\max}=10$;
- dynamic correlation coefficients $\delta_\alpha=\delta_\beta=0.2$.

An external disturbance is a meander with a frequency 12.5 radian/sec and amplitude 0.5 V.

4.3. RESULTS

4.3.1. LOW TEST FREQUENCY SEARCH

The low test frequency $\hat{\omega}_l=1.92$ radian/sec has been determined through 4 test cycles. Elapsed time for search is 73.5 sec.

4.3.2. SELF-TUNINGS OF TEST SIGNAL

The testing frequencies have been determined by expression (11) (radian/sec):

$$\omega_1=1.922,\ \ \omega_2=10.504,\ \ \omega_3=57.105. \tag{13}$$

The results of the testing amplitudes self-tuning with frequencies (13) are given in table 1, and the results of duration self-tuning are given in the table 2.

Table 1. The results of the testing amplitudes self-tuning

Frequency, radian/sec	Number of self-tuning cycles	Final amplitude	χ
$\omega_1=1.922$	4	0.5	0.47322
$\omega_2=10.504$	2	2	0.47958
$\omega_3=57.105$	1	4	0.18725

Table 2. The results of the test signal duration self-tuning

Frequency, radian/sec	Number of self-tuning cycles	Results				
		α	β	$K_\alpha(\tau)$	$K_\beta(\tau)$	τ, sec
$\omega_1=1.922$	2	0.45201	-0.54211	0.02150	0.04407	13.08

$\omega_2 = 10.504$	32	-0.01689	-0.17467	0.00260	0.00020	229.69
$\omega_3 = 57.105$	1	-0.02060	0.00686	0.04279	0.00557	6.60

4.3.3. PLANT IDENTIFICATION

The result of the identification is the following estimate of the discrete-time transfer function:

$$\hat{W}_{PPM}(z) = \frac{0.000081425z^2 - 0.0001231z - 0.00007278}{z^3 - 2.888\,z^2 + 2.783z - 0.8961} = \frac{0.000081409(z^2 - 1.512z + 0.8938)}{(z - 0.9965)\,(z^2 - 1.891z + 0.899)}. \quad (14)$$

This transfer function corresponds to the continuous-time one:

$$\hat{W}_{PPM}(s) = \frac{0.037983(s^2 + 58.15s + 108100)}{(s + 1.762)(s^2 + 53.21s + 2232)}. \quad (15)$$

Comparing both plant transfer function (12) and estimated one (15), it can be seen that their denominators are quite near, but their numerators are differ, because of the small time constants in estimated transfer function (15). For the ease of comparing their gain-frequency and phase-frequency characteristics appear on figures 2 and 3 respectively (solid lines represent PPM and chain lines correspond to estimate (15)). Comparing the transfer functions in their frequency domain, it can be seen that their characteristics within band from 0 to 100 radian/sec are very close.

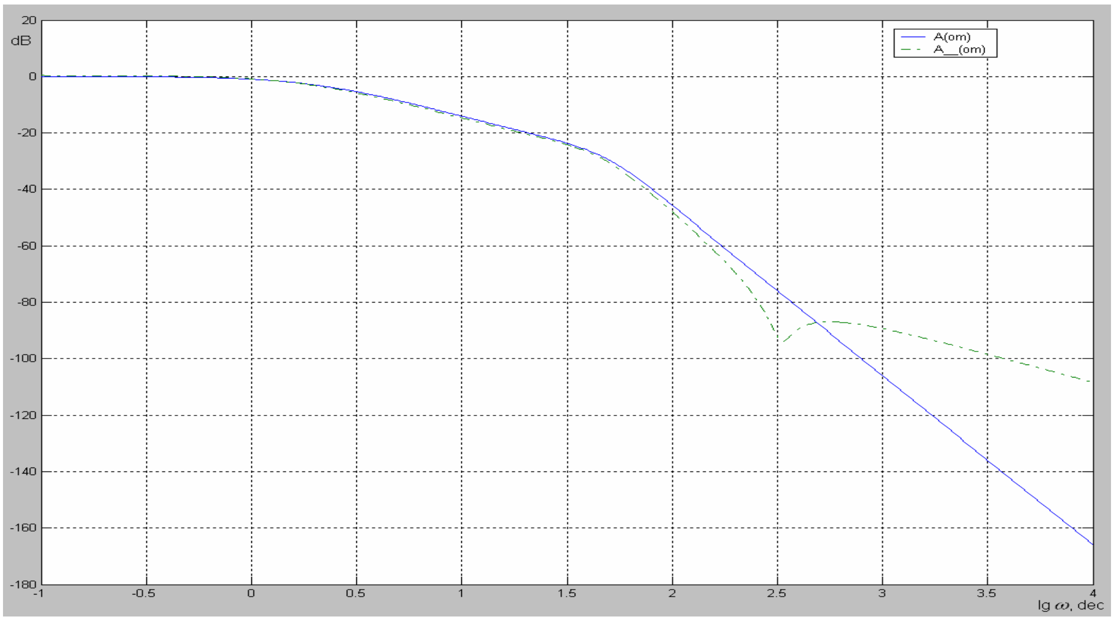

Fig. 2. Logarithmic gain-frequency characteristic of PPM and estimated

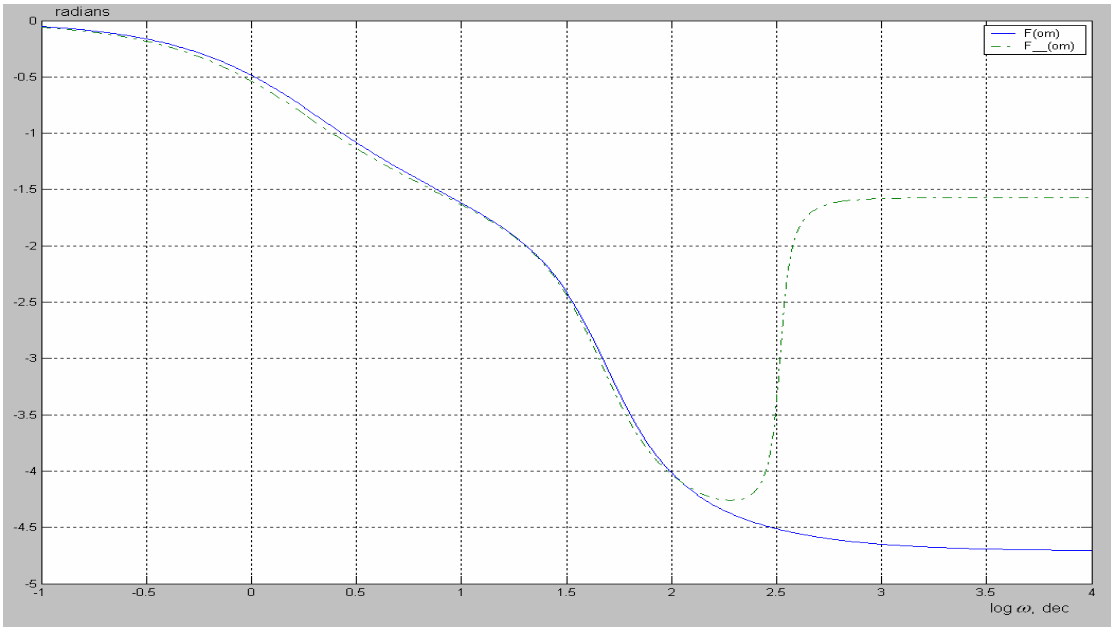

Fig. 3. Logarithmic phase-frequency characteristic of PPM and estimated

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