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Self-Tuning PID-I Controller with a new algorithm of tuning of test signal

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Abstract: The self-tuning PID/I controller (ST-PID-2) with new algorithm of tuning of test signal is proposed. This controller is based on the self-tuning PID/I controller (ST-PID-1) but have better algorithm of tuning of test signal. It is assumed that plant is described by first order with time delay. Coefficients of plant are unknown and can be changed in some isolated time moments. Self-tuning PID/I controller for the plant is proposed in the presence of unknown-but-bounded external disturbances. The test signal with sum of two harmonics is used for identification of plant coefficients.To guarantee the stability of the closed-loop system the special switching technique between I- and PID-controllers is used. The I-controller is selected to provide stability of the plant independently on its mode. The obtained results are supported by experimental applications.

Keywords: adaptive control, PID control, I-controller, time delay, frequency identification, unknown bounded disturbance.

1. INTRODUCTION

Proportional-Integral-Differential (PID) controllers are widely used in industry because they are simple and effective. Self-tuning PID controllers are designed for plants with time-varying coefficients. Different methods of the identification are used for plant identification. These methods allow to estimate plant's parameters and use them for adjusting of PID controller. However, the identification is complicated because of external disturbances. Step response methods (Ziegler J. B. and Nichols N. B. [1942]) are used when the external disturbance is absent. In this case, it is also possible to use relay methods (Ziegler J. B. and Nichols N. B. [1942], Astrom K. J. and Hagglund T. [1984], Hang C.C. et al. [1993], Astrom K. J. and Hagglund T. [2006]). However, the relay method is not suitable because it breaks the normal mode of the plant. In papers Takao Sato and Akira Inoue [2005] and Takao Sato and Koichi Kameoka [2008], the least-squares method (LSM) is used for determination of coefficients of the plant. In the paper Ho W.K. et al. [1996], the same method is used in conjunction with band-pass filter. This method allows better detection of coefficients of the plant. However, LSM is not applicable when the external disturbance is an unknown bounded function. It is easy to find an external disturbance for LSM which gives impermissible identification errors.

In paper Alexandrov A.G. and Palenov M.V. [2011] the self-tuning PID/I controller is proposed. Coefficients of plant are unknown and they change in sufficiently seldom time moments. The identification is complicated due to external disturbances. The finite-frequency identification method is used for identification (A.G. [1994], A.G. [1999]). The plant is excited by a test signal which is a sum of two harmonics. Amplitudes of the test signal is adjusted so that the level of distortions introduced in the plant's output does not exceed the specified limit. Fourier's filter is used. Identification error for given filtration time depends on the choice of test signal frequencies (see, for example, A.G. [2005]). The frequencies must be chosen in such way that minimizes an identification error of the given filtration time.

The results of identification are used for design of PID controller based on concept of Internal Model Principle (Antonio [2002]). This controller compensates the time constant of the plant, so the system performance is determined by the given parameter of synthesis of controller and time delay in the control channel. PID controller also provides high amplitude and phase margins.

The closed loop system with PID controller may lose stability because of change of plant coefficients. In this case, the plant is closed by I-controller instead of PID controller. I-controller can provide stability for a large range of plant's coefficients and it also allows tracking a reference signal without static error. On the other hand, this controller can not provide fast reference tracking performance. So the plant is closed by I-controller only when the closed loop with PID controller loses stability. Thus, the loop of the system is not broken. It allows to identify the plant.

In this paper a new algorithm of test signal amplitudes tuning proposed. The proposed algorithm is faster and more convenient than the previous.

The paper is organized as follows. In the next section the problem statement and the basic assumptions are presented. The Section 3 is devoted to an identification problem of the plant by the finite-frequency method. A new algorithm of tuning of amplitudes are proposed in Section 4. In Section 5 an expression for coefficient of I-controller is given and algorithms of self-tuning of PID/I controller are proposed. In Section 6, real self-tuning controller named *ST-PID-2*, which implement on the basis of these algorithms, is described. Experimental investigations of controller are given.

2. PROBLEM STATEMENT

Consider a plant described by equation

$$T^{[i]}\dot{y}(t) + y(t) = k_p^{[i]}u(t-\tau^{[i]}) + f(t), \qquad (1)$$
$$t^{[i]} \le t < t^{[i+1]}, \quad i = 1,2,...,N,$$

where $y(t)$ and $u(t)$ are output and input of the plant respectively, $f(t)$ is an unknown-but-bounded external disturbance ($|f(t)| \le f^*$), i is the number of the plant's mode ($i = 1,2,...N$). Coefficients $k_p^{[i]}$, $T^{[i]}$, $\tau^{[i]}$ are unknown numbers, they change in known (for simplicity) time moments $t^{[1]}$, $t^{[2]}$, ..., $t^{[N]}$, and they are constant in each i-th mode

$$t^{[i]} \le t < t^{[i+1]}, \quad i = 1,2,...,N. \qquad (2)$$

The possible values of plant's coefficients lie into intervals

$$\underline{k_p} \le k_p^{[i]} \le \overline{k_p}, \;\; \underline{T} \le T^{[i]} \le \overline{T}, \;\; \underline{\tau} \le \tau^{[i]} \le \overline{\tau}, \qquad (3)$$
$$i = 1,2,...,N,$$

where lower $(\underline{k_p}, \underline{T}, \underline{\tau})$ and upper $(\overline{k_p}, \overline{T}, \overline{\tau})$ bounds are given positive numbers.

The PID controller is

$$g^{[i]}\dot{u}(t) + u(t) = k_c^{[i]}\varepsilon^{[i]}(t) + k_i^{[i]}\int_0^t \varepsilon^{[i]}(\tilde{t})d\tilde{t} + k_d^{[i]}\dot{\varepsilon}^{[i]}(t),$$
$$t_{st}^{[i]} \le t < t_{st}^{[i+1]}, \;\; t^{[i]} \le t_{st}^{[i]} < t^{[i+1]}, \;\; i = 1,2,...,N. \qquad (4)$$

$$\varepsilon^{[i]}(t) = y_{sp}^{[i]} - y(t) + v^{[i]}(t), \qquad (5)$$

where $g^{[i]}$, $k_c^{[i]}$, $k_i^{[i]}$, $k_d^{[i]}$ are coefficients of the PID controller, they are changing in the time moments $t_{st}^{[i]}$, $\varepsilon(t)$ is the tracking error, $v(t)$ is the test signal, $y_{sp}^{[i]}$ is the reference signal.

The reference signal $y_{sp}^{[i]}$ is known and constant in each i-th mode.

Modes of plant and PID controller are illustrated in picture 1.

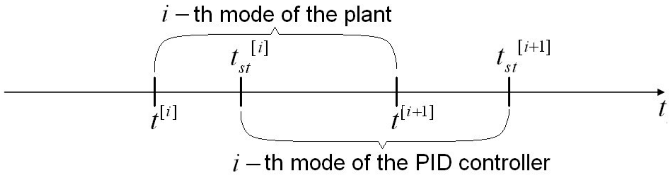

Fig. 1. Time intervals of plant and PID controller

Note, there does not exist a PID controller with time invariant coefficients that could provide stability for plant in each mode.

Tracking error (5) must satisfy the following condition:

$$|\varepsilon^{[i]}(t)| = |\varepsilon^{[i]*}(t)| + |\xi^{[i]}(t)|, \;\; t \ge t_{st}^{[i]}, \;\; i = 1,2,...,N, \qquad (6)$$

where $|\varepsilon^{[i]*}(t)|$ is the achieved tracking error (ideal tracking error) in case when the plant in i-th mode is known. Values $|\xi^{[i]}(t)|$ must satisfy the condition:

$$|\xi^{[i]}(t)| < q|\varepsilon^{[i]*}(t)|, \quad i = 1,2,...,N, \qquad (7)$$

where q is the sufficiently small positive number.

Coefficients of the PID controller (4) are calculated through the following expressions (Antonio [2002])

$$k_c^{[i]} = \frac{2T^{[i]} + \tau^{[i]}}{2k_p^{[i]}(\lambda^{[i]} + \tau^{[i]})}, \;\; k_i^{[i]} = \frac{1}{k_p^{[i]}(\lambda^{[i]} + \tau^{[i]})},$$
$$k_d^{[i]} = \frac{T^{[i]}\tau^{[i]}}{2k_p^{[i]}(\lambda^{[i]} + \tau^{[i]})}, \;\; g^{[i]} = \frac{\lambda^{[i]}\tau^{[i]}}{2(\lambda^{[i]} + \tau^{[i]})}, \qquad (8)$$
$$i = 1,2,...,N,$$

where $\lambda^{[i]}$ is the design parameter. It is chosen as $\lambda^{[i]} = \frac{T^{[i]}}{\o}$, where $\o = 2 \div 4$ (Antonio [2002], Astrom K. J. and Hagglund T. [2006]).

The closed loop system (1), (4), (5) is approximately described by the following differential equation

$$\lambda^{[i]}\dot{y}(t) + y(t) = y_{sp}(t - \tau^{[i]}). \qquad (9)$$

The influence of test signal $v^{[i]}(t)$ to the input and the output of plant are bounded in each mode

$$y_v^{[i]} = \max|y(t) - \overline{y}(t)|, \;\; u_v^{[i]} = \max|u(t) - \overline{u}(t)|, \qquad (10)$$
$$t^{[i]} \le t < t_{st}^{[i]}, \;\; i = 1,2,...,N,$$

where $\overline{y}(t)$ and $\overline{u}(t)$ are output and input of the plant without test signal ($v(t) = 0$).

Bounds (10) of the influence of test signal are satisfied the following condition

$$|y_v^{[i]}| \le y_-, \;\; |u_v^{[i]}| \le u_-, \;\; i = 1,2,...,N, \qquad (11)$$

where y^* and u^* are given positive numbers.

The problem is to find coefficients of PID controller in each i-th mode such that the conditions (6) and (11) are satisfied.

Assumptions.

A) Operation time of the plant $\Delta_i = t^{[i+1]} - t^{[i]}$ is sufficiently large such that the following condition is satisfactory $\Delta_i > t^{[i+1]} - t^{[i]}$ (etc. $t^{[i+1]} - t^{[i]}$ $(i = 1,2,...,N)$ (etc. $t^{[i+1]} - t^{[i]}$ $(i = 1,2,...,N)$ is the self-tuning time).

B) In order to provide identification accuracy of the plant is sufficiently follow inequalities

$$\left|\frac{2}{\rho_k\Delta_i}\int_{t^{[i]}}^{t_{st}^{[i]}} \overline{y}\sin(\omega_k t)dt\right| \le l_\alpha, \;\; \left|\frac{2}{\rho_k\Delta_i}\int_{t^{[i]}}^{t_{st}^{[i]}} \overline{y}\cos(\omega_k t)dt\right| \le l_\beta,$$
$$k = 1,2, \;\; i = 1,2,...,N, \qquad (12)$$

where l_α and l_β are sufficiently small numbers.

If $f(t)$ is expanded in a Fourier series, then the inequality (12) means that the amplitudes of harmonics of $f(t)$ are sufficiently small at frequencies ω_1 and ω_2.

3. IDENTIFICATION OF THE PLANT

The problem of finding of PID controller coefficients is reduced to identification coefficients of plant (1) in each

i-th mode. PID parameters are calculated through of plant coefficients by (8). So it is important to identify the coefficients of the plant more accurately because the effectiveness of self-tuning of PID-controller depends on the accuracy of the identification results.

3.1 Finite-frequency identification

There are some difficulties in the identification of plant:

a) reference signal is often a constant function therefore the input signal has not enough harmonics (L. [1987]);

b) often, the external disturbance is an unknown-but-bounded function;

c) the plant must be identified in the closed loop (1), (4), (5).

The above problems can be solved by using the finite-frequency identification method. In accordance with this method numbers

$$\alpha_k^{[i]} = \mathrm{Re}\; w_p^{[i]}(j\omega_k), \;\; \beta_k^{[i]} = \mathrm{Im}\; w_p^{[i]}(j\omega_k), \;\; k = 1, 2, \quad (13)$$

where

$$w_p^{[i]}(s) = \frac{k_p^{[i]} e^{-\tau^{[i]} s}}{T^{[i]} s + 1}, \;\; i = 1, 2, ..., N, \quad (14)$$

are called *frequency domain parameters* (FDP) (A.G. [1994]).

The FDP estimates are determined experimentally as follows: after the closed loop system is excited by the test signal

$$v^{[i]}(t) = \rho_1^{[i]} \sin \omega_1 t + \rho_2^{[i]} \sin \omega_2 t, \quad (15)$$

where $\rho_k^{[i]}$ and test frequencies ω_k $(k = 1, 2)$ are specified positive numbers, test frequencies are multiples of each other $\omega_2 = \mu\omega_1$ $(1 < \mu < \infty$, μ is integer), plant's input $u(t)$ and output $y(t)$ are fed to the Fourier filters, whose outputs give the following estimates

$$\begin{aligned}
\hat{\alpha}_{yk}^{[i]} &= \alpha_{yk}^{[i]}(\bar{t}^{[i]}) = \frac{2}{\rho_k^{[i]} \bar{t}^{[i]}} \int_{t_F^{[i]}}^{t_F^{[i]}+\bar{t}^{[i]}} y(t) \sin \omega_k t dt, \\
\hat{\beta}_{yk}^{[i]} &= \beta_{yk}^{[i]}(\bar{t}^{[i]}) = \frac{2}{\rho_k^{[i]} \bar{t}^{[i]}} \int_{t_F^{[i]}}^{t_F^{[i]}+\bar{t}^{[i]}} y(t) \cos \omega_k t dt, \\
\hat{\alpha}_{uk}^{[i]} &= \alpha_{uk}^{[i]}(\bar{t}^{[i]}) = \frac{2}{\rho_k^{[i]} \bar{t}^{[i]}} \int_{t_F^{[i]}}^{t_F^{[i]}+\bar{t}^{[i]}} u(t) \sin \omega_k t dt, \\
\hat{\beta}_{uk}^{[i]} &= \beta_{uk}^{[i]}(\bar{t}^{[i]}) = \frac{2}{\rho_k^{[i]} \bar{t}^{[i]}} \int_{t_F^{[i]}}^{t_F^{[i]}+\bar{t}^{[i]}} u(t) \cos \omega_k t dt, \\
& k = 1, 2, \;\; i = 1, 2, ..., N,
\end{aligned} \quad (16)$$

where $\bar{t}^{[i]}$ is a filtering time and $t_F^{[i]}$ is the initial instant for filtering. Numbers $\bar{t}^{[i]}$ and $t_F^{[i]}$ are multiples of a base period $T_b = \frac{2\pi}{\omega_1}$ and satisfied the inequality $t^{[i]} < t_F^{[i]} + t^{[i]} < t^{[i+1]}$. The numbers $\hat{\alpha}_{yk}^{[i]}$, $\hat{\beta}_{yk}^{[i]}$, $\hat{\alpha}_{uk}^{[i]}$, $\hat{\beta}_{uk}^{[i]}$ $(k = 1, 2)$ allow us to estimate the plant model coefficients.

If the disturbance $f(t)$ and reference signal $y_{sp}(t)$ are strongly FF-filterability (A.G. [2005]), this mean that disturbance $f(t)$ and reference signal $y_{sp}(t)$ does not contain test frequencies ω_1, ω_2, then

$$\begin{aligned}
&\lim_{\bar{t}^{[i]} \to \infty} \alpha_{yk}^{[i]}(\bar{t}^{[i]}) = \alpha_{yk}^{[i]}, \;\; \lim_{\bar{t}^{[i]} \to \infty} \beta_{yk}^{[i]}(\bar{t}^{[i]}) = \beta_{yk}^{[i]}, \\
&\lim_{\bar{t}^{[i]} \to \infty} \alpha_{uk}^{[i]}(\bar{t}^{[i]}) = \alpha_{uk}^{[i]}, \;\; \lim_{\bar{t}^{[i]} \to \infty} \beta_{uk}^{[i]}(\bar{t}^{[i]}) = \beta_{uk}^{[i]}, \\
&\qquad k = 1, 2, \;\; i = 1, 2, ..., N,
\end{aligned}$$

where $\alpha_{yk}^{[i]}$, $\beta_{yk}^{[i]}$, $\alpha_{uk}^{[i]}$, $\beta_{uk}^{[i]}$ $(k = 1, 2)$ FDP of the closed loop system (A.G. [1998]):

$$\begin{aligned}
\alpha_{yk}^{[i]} + j\beta_{yk}^{[i]} &= \frac{W_c^{[i]}(j\omega_k) W_p^{[i]}(j\omega_k)}{1 + W_c^{[i]}(j\omega_k) W_p^{[i]}(j\omega_k)}, \\
\alpha_{uk}^{[i]} + j\beta_{uk}^{[i]} &= \frac{W_c^{[i]}(j\omega_k)}{1 + W_c^{[i]}(j\omega_k) W_p^{[i]}(j\omega_k)}, \\
& k = 1, 2, \;\; i = 1, 2, ..., N,
\end{aligned} \quad (17)$$

where $w_c^{[i]}(j\omega_k)$ is the frequency transfer function of the PID controller. Conditions of FF-filterability can be examined by experiment by using the Fourier filter (16) without test signal $(v^{[i]}(t) = 0)$. Condition of strongly FF-filterability is satisfied when outputs of filter are zero (See A.G. [2005]). If external disturbance $f(t)$ and reference signal $y_{sp}(t)$ are only FF-filterability, this means that the signals $f(t)$ and $y_{sp}(t)$ contain frequencies ω_1 and ω_2 with sufficiently small amplitude, then following conditions are true

$$\begin{aligned}
&\lim_{\bar{t}^{[i]} \to \infty} \alpha_{yk}^{[i]}(\bar{t}^{[i]}) = \alpha_{yk}^{[i]} + \zeta_{\alpha yk}, \\
&\lim_{\bar{t}^{[i]} \to \infty} \beta_{yk}^{[i]}(\bar{t}^{[i]}) = \beta_{yk}^{[i]} + \zeta_{\beta yk}, \\
&\lim_{\bar{t}^{[i]} \to \infty} \alpha_{uk}^{[i]}(\bar{t}^{[i]}) = \alpha_{uk}^{[i]} + \zeta_{\alpha uk}, \\
&\lim_{\bar{t}^{[i]} \to \infty} \beta_{uk}^{[i]}(\bar{t}^{[i]}) = \beta_{uk}^{[i]} + \zeta_{\beta uk}, \\
&\qquad k = 1, 2, \;\; i = 1, 2, ..., N,
\end{aligned}$$

where $\zeta_{\alpha yk}$, $\zeta_{\beta yk}$, $\zeta_{\alpha uk}$ and $\zeta_{\beta uk}$ $(k = 1, 2)$ are sufficiently small numbers.

Numbers $\alpha_{yk}^{[i]}$, $\beta_{yk}^{[i]}$, $\alpha_{uk}^{[i]}$, $\beta_{uk}^{[i]}$ $(k = 1, 2)$ are related with FDP $\alpha_k^{[i]}$, $\beta_k^{[i]}$ $(k = 1, 2)$ as follows (Alexandrov A.G. and Palenov M.V. [2011])

$$\alpha_k^{[i]} = \frac{\alpha_{yk}^{[i]} \alpha_{uk}^{[i]} + \beta_{yk}^{[i]} \beta_{uk}^{[i]}}{(\alpha_{uk}^{[i]})^2 + (\beta_{uk}^{[i]})^2}, \;\; \beta_k^{[i]} = \frac{-\alpha_{yk}^{[i]} \beta_{uk}^{[i]} + \beta_{yk}^{[i]} \alpha_{uk}^{[i]}}{(\alpha_{uk}^{[i]})^2 + (\beta_{uk}^{[i]})^2}, \quad (18)$$

$$k = 1, 2, \;\; i = 1, 2, ..., N.$$

Coefficients of the plant (1) and FDP (13) are related by following expressions (for simplicity, the index $[i]$ omitted)

$$T^2 = \frac{(\alpha_2^2 + \beta_2^2) - (\alpha_1^2 + \beta_1^2)}{\omega_1^2(\alpha_1^2 + \beta_1^2) - \omega_2^2(\alpha_2^2 + \beta_2^2)}, \quad (19.a)$$

$$k_p^2 = (\alpha_2^2 + \beta_2^2)(T^2\omega_2^2 + 1), \quad (19.b)$$

$$\tau = \frac{1}{\omega_1} atan \frac{T\omega_1\alpha_1 + \beta_1}{T\omega_1\beta_1 - \alpha_1} \quad (19.c)$$

$$\omega_1 \tau < \frac{\pi}{2} \quad (19.d)$$

4. TUNING OF THE TEST SIGNAL

For good identification results frequencies of the plant must be choose to close to own frequencies of the plant. There are some discussion in the Alexandrov A.G. and Palenov M.V. [2011], then choose the frequencies of the test signal as follows

$$\omega_1 = \frac{1}{2\overline{T}}, \ \ \omega_2 = 2\omega_1. \tag{20}$$

Let's consider the steady state of a closed system with a test signal (15). Input and output of the plant in the absence of external perturbation ($f(t) = 0$) have the form

$$\begin{aligned} u(t) &= \sum_{k=1}^{2} \rho_k(\alpha_{uk}^{[i]} \sin\omega_k t + \beta_{uk}^{[i]} \cos\omega_k t) = \\ &= \sum_{k=1}^{2} \rho_{uk} \sin(\omega_k t + \phi_{uk}), \\ y(t) &= \sum_{k=1}^{2} \rho_k(\alpha_{yk}^{[i]} \sin\omega_k t + \beta_{yk}^{[i]} \cos\omega_k t) = \\ &= \sum_{k=1}^{2} \rho_{yk} \sin(\omega_k t + \phi_{yk}), \end{aligned}$$

where

$$\begin{aligned} \rho_{uk} &= \rho_k\sqrt{(\alpha_{uk}^{[i]})^2 + (\beta_{uk}^{[i]})^2}, \\ \rho_{yk} &= \rho_k\sqrt{(\alpha_{yk}^{[i]})^2 + (\beta_{yk}^{[i]})^2}, \end{aligned} \quad k = 1, 2, \tag{21}$$

are amplitudes of input and output of the plant (See (17)).

If we take the

$$\rho_{u1} = \rho_{u2} = \frac{1}{2}u_- \text{ and } \rho_{y1} = \rho_{y2} = \frac{1}{2}y_-,$$

then with take into account (21) we obtain

$$\rho_k^{[i]} = \frac{1}{2}\min\left\{ \frac{u_-}{\sqrt{(\alpha_{uk}^{[i]})^2 + (\beta_{uk}^{[i]})^2}}, \frac{y_-}{\sqrt{(\alpha_{yk}^{[i]})^2 + (\beta_{yk}^{[i]})^2}} \right\}, \tag{22}$$

$$k = 1, 2.$$

In the case of external disturbance ($f(t) \neq 0$) the values of the FDP of the closed system used in (22), are estimated using a Fourier filter (16). Then the expression (22) takes the form

$$\rho_k^{[i]} = \frac{1}{2}\min\left\{ \frac{u_-}{\sqrt{\left(\hat{\alpha}_{uk}^{[i]}\right)^2 + \left(\hat{\beta}_{uk}^{[i]}\right)^2}}, \frac{y_-}{\sqrt{\left(\hat{\alpha}_{yk}^{[i]}\right)^2 + \left(\hat{\beta}_{yk}^{[i]}\right)^2}} \right\}. \tag{23}$$

Amplitudes are self-tuned by the following algorithm. *Algorithm 3.1*

(1) Take sufficiently small amplitudes ρ_k ($k = 1, 2$) of the test signal, such that conditions (11) are satisfied, and calculate estimates of the FDP of the closed system using the Fourier filter (16).

(2) Find the amplitudes of the test signal using the formula (23).

5. I-CONTROLLER AND SELF-TUNING ALGORITHM OF PID/I CONTROLLER

5.1 I-controller

The plant in $(i + 1)$-th mode closed by PID-controller, which designed for the plant of i-th mode, may lose stability. In this case, for identification, the plant is closed by I-controller. I-controller (it follows from (4) with $g = k_c = k_d = 0$) is

$$u(t) = k_i \int_{t_0}^{t} \varepsilon(t)dt \tag{24}$$

where k_i is constant for all modes.

The closed loop system (1), (24), (5) is stable if

$$0 < k_i < \frac{l_m}{\overline{k}_p}, \tag{25}$$

where

$$l_m = \min_{\substack{\underline{\tau}\leq\tau\leq\overline{\tau} \\ \underline{T}\leq T\leq\overline{T}}} \frac{\omega_u}{\sin\omega_u \tau} \tag{26}$$

under conditions

$$T\omega_u \sin\omega_u\tau = \cos\omega_u\tau, \tag{27}$$

$$0 < \omega_u < \frac{\pi}{2\tau}. \tag{28}$$

5.2 Algorithm of the Self-tuning PID/I controller

1) Close the plant by I-controller (24) in the first mode. Frequencies of test signal are calculated by using (20);

2) Find amplitudes of test signal are self-tuned by using *Algorithm 3.1*;

3) Identify the plant in the closed loop system with I- (or PID-controller): a) turn on the test signal (15) and feed input and output of the plant to the Fourier's filter (16) which output for a given value $\bar{t}$ (or if (**??**) is satisfied) gives the estimates of FDP (18); b) Use (19.a)-(19.c), substituting estimates of FDP, for calculation of plant's coefficients estimates and then turn off the test signal ($v(t) = 0$);

4) Coefficients of PID-controller for identified plant are calculated by using (8). Then the plant is closed by this PID controller.

5) There are two variants into the next mode of plant: a) if condition $|y(t)| \leq y^*$ is satisfied (the closed loop system is stable) then go to operation 2); b) otherwise, if the closed loop system loses stability ($|y(t)| > y^*$) then go to operation 1).

6. EXPERIMENTAL RESULTS

6.1 Experimental setup FM-2

Experimental setup FM-2 is the setup for investigations of adaptive controllers in a semi-industrial environment. This setup includes an industrial controller WinCon-8341 and industrial computer Athena, which interact with each other through embedded DAC and ADC converters. Plant simulator is implemented in the industrial computer Athena. Self-tuning PID-I controller, called ST-PID-2, is implemented in the industrial controller WinCon-8341.

6.2 Results of experiments

Transfer function of the plant have a follow form:

$$w_p(s) = \frac{k_p^{[i]} e^{-\tau^{[i]} s}}{(T^{[i]}s+1)(T_1^* s+1)(T_2^* s+1)}, \quad i = 1, 2, ..., N, \qquad (29)$$

where T_1^* and T_2^* are unmodeled dynamics $T_1^* \le T_2^* < T$ ($T_1^* = 0.2$ sec and $T_2^* = 0.3$ sec).

The plant coefficients belong to intervals

$$\underline{k_p} = 0.1, \ \ \overline{k_p} = 4, \ \ \underline{T} = 1, \ \ \overline{T} = 8, \ \ \underline{\tau} = 0.1, \ \ \overline{\tau} = 2.$$

They are changing in each i-th mode according with table 1.

Table 1. Coefficients of the plant

	1	2	3	4	5	6	7	8
k_p	3.51	2.73	2.16	1.05	1.49	3.97	3.89	2.29
T	3.22	1.49	2.20	6.90	2.63	3.69	2.53	6.13
τ	0.61	0.47	1.75	1.16	1.33	0.38	1.25	1.27

Duration of an each mode is 1400 second. External disturbance is $f(t) = 0.5\text{sign}[\sin 2.1t]$.

Test frequencies are $\omega_1 = 0.0625$ rad/s and $\omega_2 = 0.1250$ rad/s. Amplitudes of the test signal are tuned with $y_- = 1$ and $u_- = 2$. PID controller designed by (8) with $\lambda^{[i]} = \frac{\hat{T}^{[i]}}{4}$ ($i = 1, 2, ..., 8$). The identification stops when the relative identification error is $\theta = 0.02$.

Output of the system is showed in a picture 2. The function $|\xi^{[i]}(t)|$ from (6) ($|\xi^{[i]}(t)| = |\varepsilon^{[i]}(t)| - |\varepsilon^{[i]*}(t)|$) is shown at 3. I-controller is connecting in the 3-th and 5-th modes. In picture 4 shown 5-th mode. The function $|\xi^{[i]}(t)|$ in 6-th mode is shown in 5.

Notations are used in all figures: gray vertical dash line denote time moments $t^{[i]}$, gray vertical dash dot line denote time moments $t_{st}^{[i]}$.

Estimates of plant's coefficients are shown in table 2, where gray rows are shows modes when I-controller connected.

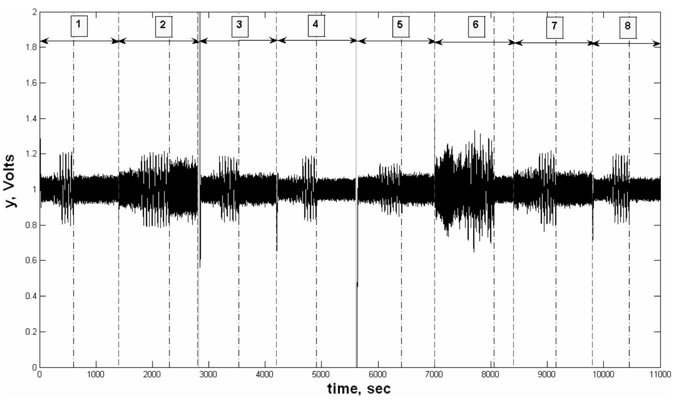

Fig. 2. Output of the system

From figures 2, 3, 4 and 5 we can conclude that purpose (6) is satisfied.

7. CONCLUSION

In this paper, a new technique of adaptive control of the multi-mode first order plant with time delay has been presented. It is based on two-frequencies identification of the plant and uses PID- and I-controllers. The new

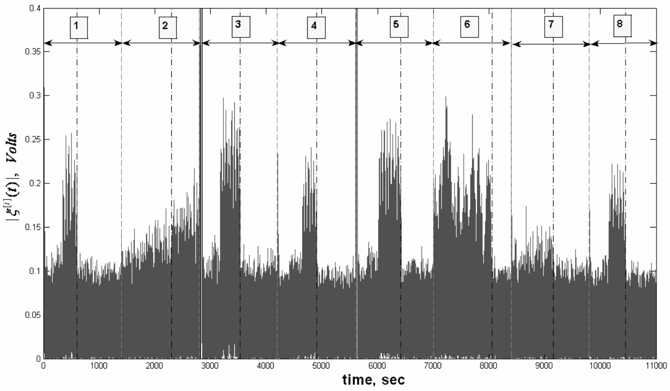

Fig. 3. Function $|\xi^{[i]}(t)|$ in all eight modes

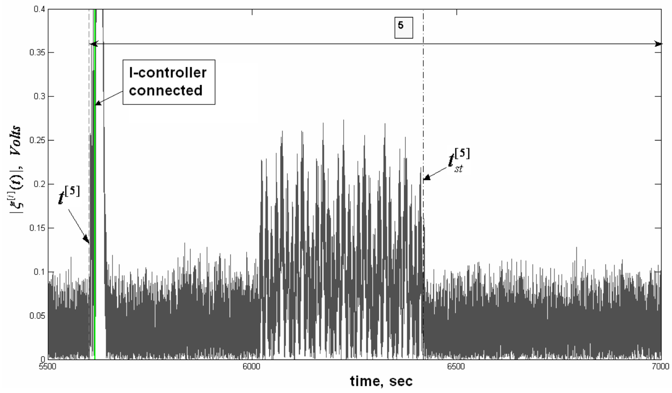

Fig. 4. Function $|\xi^{[i]}(t)|$ in 5-h mode

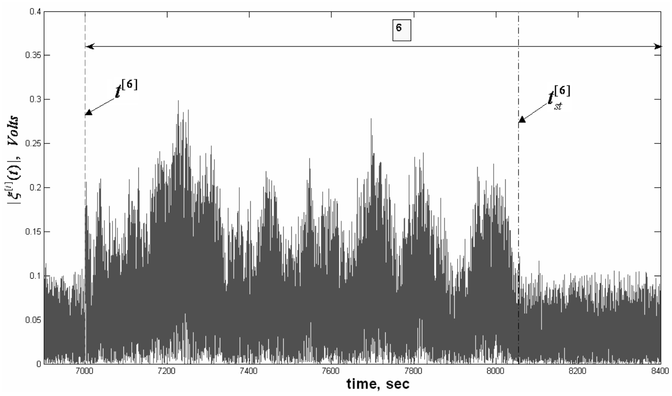

Fig. 5. Function $|\xi^{[i]}(t)|$ in 6-h mode

Table 2. Results of experiment 2

Mode number	k_p	$\hat{k_p}$	T, c	$\hat{T}$	τ, c	$\hat{\tau}$
1	**3.51**	**3.42**	**3.22**	**2.86**	**0.61**	**0.96**
2	2.73	2.78	1.49	1.71	0.47	0.20
3	**2.16**	**2.15**	**2.20**	**2.35**	**1.75**	**1.42**
4	1.05	1.09	6.90	7.32	1.16	0.92
5	**1.49**	**1.45**	**2.63**	**2.60**	**1.33**	**1.26**
6	3.97	3.95	3.69	3.72	0.38	0.45
7	3.89	3.86	2.53	2.65	1.25	1.40
8	2.29	2.33	6.13	6.14	1.27	1.26

algorithm of test signal amplitudes tuning proposed. And shown that it works.

REFERENCES

Alexandrov A.G. Finite-frequency method of identification. *Proceeding of 10th IFAC Symposium on System Identification, Preprints*, vol. 2:p.p. 523–527, 1994.

Alexandrov A.G. Frequency adaptive control of stable plant in the presence of bounded disturbance. *IFAC Workshop Adaptive Systems in Control and Signal Processing, Preprints*, pages p.p. 94–99, 1998.

Alexandrov A.G. Finite-frequency identification and model validation of stable plant. *Proceeding of 14th World Congress of IFAC, Preprints*, vol. H:p.p. 295–301, 1999.

Alexandrov A.G. Finite-frequency identification: self-tuning of test signal. *Proceeding of 16th World Congress of IFAC, Preprints*, pages p.p. 295–301, 2005.

Alexandrov A.G. and Palenov M.V. Self-tuning pid-i controller. *Preprints of the 18th IFAC World Congress*, pages p.p. 3635–3640, 2011.

Visioli Antonio. Improving the load disturbance rejection performance of imc-tuned pid controllers. *Proceeding of 15th World Congress of IFAC, Preprints*, 2002.

Astrom K. J. and Hagglund T. Automatic tuning of simple regulators with specifications on phase and amplitude margins. *Automatica*, 20 No. 5:645–651, 1984.

Astrom K. J. and Hagglund T. *Advanced PID Control.* ISA, Research Triangle Park, North Carolina, 2006.

Hang C.C., Astrom K.J., and Ho W.K. Relay auto-tuning in the presence of static load disturbance. *Automatica*, 29:2:563–564, 1993.

Ho W.K., Hang C.C., Wojsznis W., and Tao Q. H. Frequency domain approach to self-tuning pid control. *Control Engineering Practice*, 4 No. 6:807–813, 1996.

Ljung L. *System Identification. Theory for the User.* Englewood Cliffs: Prentice-Hall, 1987.

Takao Sato and Akira Inoue. Future reference trajectory improvement in self-tuning i-pd controller based on generalized predictive control law. *Proceeding of 16th World Congress of IFAC, Preprints*, 2005.

Takao Sato and Koichi Kameoka. Self-tuning type-2 pid control system and its application. *Proceeding of 17th World Congress of IFAC, Preprints*, 2008.

Ziegler J. B. and Nichols N. B. Optimum settings for automatic controllers. *ASME Transactions*, 64:759–768, 1942.