

## Package "Automatica": New Opportunities

Albert Alexandrov\* Dmitriy Shatov\*\*

\* *Institute of Control Sciences, Profsoyuznaya, 65, Moscow, Russia  
(e-mail: alex7@ipu.ru)*

\*\* *Institute of Control Sciences, Profsoyuznaya, 65, Moscow, Russia  
(e-mail: dvshatov@gmail.com)*

---

**Abstract:** Package "Automatica" is intended for analysis and development of control algorithms for plants, considering parametric and structural uncertainty and influence of unknown-but-bounded disturbance. The package is intended for scientists and developers of real-world control systems. The goal of an engineer is to guarantee given steady-state error tolerance for each of the controlled variables while the disturbance is unknown but bounded and the uncertain plant parameters change their values slowly. Consequently the periodic identification of the plant model and appropriate redesign of the controller are required. The structure of the package is specially oriented for such kind of problems. The basis of this structure consists of so called directives that solve some defined class of problem.

*Keywords:* software, multivariable plant, controller design, identification, adaptive control, unknown-but-bounded disturbance.

---

### 1. INTRODUCTION

Package "Automatica" is intended for analysis, development and synthesis of plant controllers, their identification and adaptation. Plants may have parametric and structural uncertainty and work under unknown external disturbance. Measurement noises can also be accounted for. Package consists of functions and so called directives for solving real-world problems. Programs are built from functions and are used to find solutions to defined classes of problems (such programs are further referred to as "directives"). Each directive contains software modules, that are responsible for user interface and corresponding calculations. At the end of its execution directives generate final report that can have different level of detail (summary, average or full). Package "Automatica" contains three groups of directives: controller design, finite-frequency identification, frequency-adaptive control. It has a common interface for displaying and running all groups of directives.

The software package is oriented towards two kinds of users. The first group of users includes researchers with profound knowledge of control theory and deep understanding of their objective-specific problems. The package provides a number of functions, that users can include in their own programs. The second targeted user group consists of control system developers. The aim of control system engineer is developing a complete working system by the specified deadline, therefore their capability of directly participating in software development is usually rather limited. Besides, profound knowledge of control theory is crucial for developing custom directives. An engineer can just select the directive, which solves his specific problem from the list of available directives in the package, and then enter the description of his objective. While analyzing the output report of the directive, an engineer can make a

decision on acceptability of the designed controller without deep understanding of underlying control theory.

This package is an extension of our earlier developed software. It is based on two our previous software products: GAMMA-1PC Alexandrov (1997) and ADAPLAB-3 Alexandrov et al. (2009). The functions and directives of package GAMMA-1PC, which was oriented towards controller design for MIMO systems, were written in FORTRAN. In the new package they are rewritten in MATLAB. See MATLAB (2001). Package ADAPLAB-3 was developed for the same purposes as the package "Automatica". The difference between the packages is that ADAPLAB-3 provided self-tuning for SISO-plant, while the new package "Automatica" provides self-tuning of test signal for MIMO-plant. Introduction of test signal self-tuning is considered to be a significant improvement. Adaptive control is based on the well-known finite-frequency identification method. This method uses test signals, in which the number of harmonics does not exceed the plant order. Accurate extraction of frequencies and their corresponding amplitudes of the test signal is the main problem of this method. During the self-tuning, amplitudes of the test signal are being adjusted in such a manner, that the difference between plant's output in the presence and in the absence of test signal lies in the allowable bounds. The self-tuning of frequencies allows duration of identification to be decreased.

Facilities of the proposed package can be compared to MATLAB toolbox. Control System Toolbox is intended for users belonging to the first of the two earlier mentioned groups, who have adequate knowledge of control theory and experience in their specific area of practical control theory application (ex., aircraft, power engineering, etc.) Such specialists can use the offered broad spectrum of M-functions to develop custom solutions to their own real world control problems. This toolbox is however not con-

venient for engineers with more of a system-oriented approach. Frequency-Domain System Identification Toolbox is applicable in situations, when there is no disturbance present or disturbance can be considered to be white noise (frequency domain method, least-square identification). The method of instrumental variables Ljung (1987) is applicable for arbitrary disturbances but the self-tuning of controlled input is not provided for this method. Thereat the duration of identification may be too long. Thus this method is not applicable for adaptive control.

## 2. DIRECTIVES OF CONTROLLER DESIGN

### 2.1 Area of application

Consider asymptotically stable control system described by equations

$$\dot{x} = Ax + B_2u + B_1f, y = C_2x + \eta, z = C_1x, \quad (1)$$

$$\dot{x}_c = A_c x_c + B_c y, u = C_c x_c + D_c y \quad (2)$$

where  $x(t) \in R^n$  is the vector of the plant (1) states,  $x_c(t) \in R^{n_c}$  is the vector of the controller (2) states,  $u(t) \in R^m$  is the control signal,  $y(t) \in R^r$  is the vector of the measured variables,  $z(t) \in R^m$  is the vector of the controlled variables,  $f(t) \in R^m$  is the vector of unmeasured disturbance,  $\eta(t) \in R^r$  is the measurement noise,  $A, B_1, B_2, C_1, C_2, A_c, B_c, C_c, D_c$  are numerical matrices.

Disturbance and noise are bounded polyharmonic functions

$$f_i(t) = \sum_{k=0}^{\infty} f_{ik} \sin(\omega_k t + \varphi_k) \quad (i = \overline{1, m}, j = \overline{1, l}) \quad (3)$$

$$\eta_j(t) = \sum_{k=0}^{\infty} \eta_{jk} \sin(\tilde{\omega}_k t + \tilde{\varphi}_k)$$

where frequencies  $\omega_k$  and  $\tilde{\omega}_k$  and phases  $\varphi_k$  and  $\tilde{\varphi}_k$  ( $k = \overline{0, \infty}$ ) are unknown. The unknown amplitudes  $f_{ik}$  and  $\eta_{jk}$  ( $i = \overline{1, m}, j = \overline{1, l}, k = \overline{0, \infty}$ ) meet the conditions

$$\sum_{k=0}^{\infty} |f_{ik}(t)| \leq f_i^*, \sum_{k=0}^{\infty} |\eta_{jk}(t)| \leq \eta_j^* \quad (4)$$

$$(i = \overline{1, m}, j = \overline{1, l})$$

where  $f_i^*$  and  $\eta_j^*$  ( $i = \overline{1, m}, j = \overline{1, l}$ ) are known values. If the conditions (4) are satisfied then the following inequalities hold true

$$|f_i(t)| \leq f_i^*, |\eta_j(t)| \leq \eta_j^* \quad (i = \overline{1, m}, j = \overline{1, l}) \quad (5)$$

In particular case functions (3) are piecewise continuous functions, that can be factorized into Fourier series and meet conditions (4).

Steady-state errors of controlled variables and controls are determined like the following

$$z_{i,st} = \lim_{t \rightarrow \infty} \sup |z_i(t)| \quad (i = \overline{1, m}) \quad (6)$$

The goal of control algorithm is to ensure that

$$z_{i,st} \leq z_{i,st}^* \quad (i = \overline{1, m}) \quad (7)$$

where  $z_{i,st}^*$  ( $i = \overline{1, m}$ ) are given numbers. Control method that achieves (7) can be referred to as accurate control.

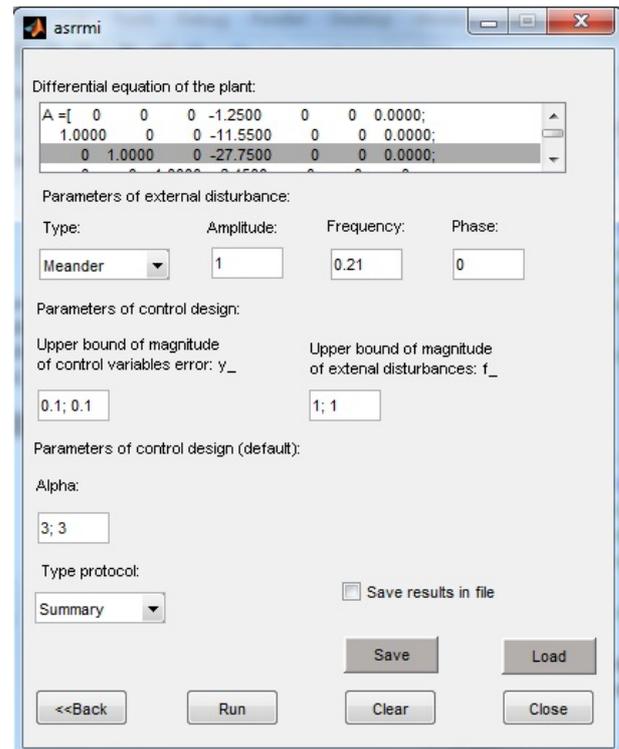


Fig. 1. Interface of the controller design directive

The aforementioned directives of controller design are intended for the finding controller matrices(2), that allow accurate control to be achieved (7) for the plant (1) under different levels of uncertainty of the plant (1) matrices.

The differential equation of the plant is entered either in the space-state form or in the arbitrary form. The arbitrary form is:

$$\left( \sum_{i=0}^{l_1} L_i s^i \right) q = \left( \sum_{i=0}^{l_2} L_i^u s^i \right) u + \left( \sum_{i=0}^{l_3} L_i^f s^i \right) f, \quad (8)$$

$$y = D^y q + \eta, z = D^z q;$$

where  $s$  is the symbol of Laplace transform under zero initial conditions,  $L_i^{(r)}$ ,  $L_j^u$ ,  $L_\mu^f$  ( $i = \overline{1, l_1}, j = \overline{1, l_2}, \mu = \overline{1, l_3}$ ),  $D^y, D^z$  are numerical matrices,  $q(t) \in R^l$  is the vector of intermediate variables.

### 2.2 Interface

The user interface for the controller design directive is shown in Fig. 1. The plant is described in state-space form (1), where

$$A = \begin{bmatrix} 0 & 0 & 0 & -1.25 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -11.55 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -27.75 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -6.45 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.33 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -13.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -11.33 & 0 \end{bmatrix} \quad (9)$$

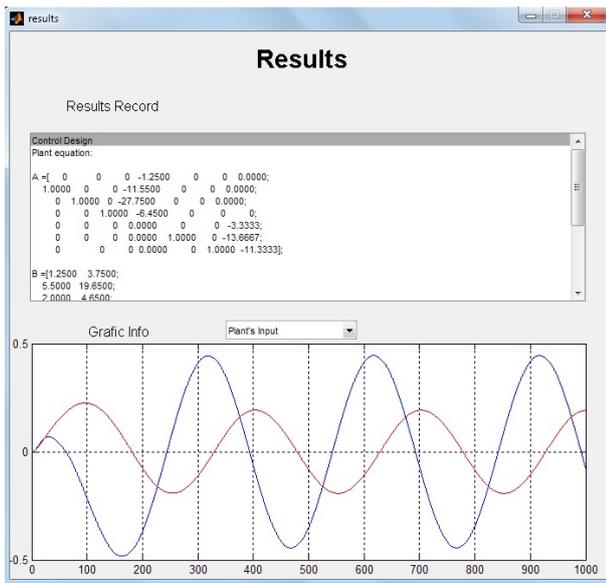


Fig. 2. Interface of directives of controller design

$$B_2 = \begin{bmatrix} 1.25 & 3.75 \\ 5.5 & 19.65 \\ 2 & 4.65 \\ 0 & 0.75 \\ 13.33 & 20 \\ 53.33 & 2 \\ 40 & 0 \end{bmatrix} \quad (10)$$

$$B_1 = \begin{bmatrix} 1.25 & 0 \\ 5.5 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 20 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \quad (11)$$

$$C_1 = C_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (13)$$

External disturbances are like the following:

$$f_1 = f_2 = \sin(2.1t); \quad (14)$$

Required steady-state errors of controlled variables and bounds of disturbances are like the following:

$$\begin{aligned} y_1^* &= y_2^* = 0.1; \\ f_1^* &= f_2^* = 1; \end{aligned} \quad (15)$$

Default parameters  $\alpha_1$  and  $\alpha_2$  both equal 3.

The results of the directive execution are shown in Fig 2. The developed controller (2) has the following matrices

$$A_c = \begin{bmatrix} 0 & 0 & 0 & -701 & 0 & 0 & 0 & -1162 \\ 1 & 0 & 0 & -2909 & 0 & 0 & 0 & -1403 \\ 0 & 1 & 0 & -2393 & 0 & 0 & 0 & -296 \\ 0 & 0 & 1 & -282 & 0 & 0 & 0 & -46 \\ 0 & 0 & 0 & -5248 & 0 & 0 & 0 & -8756 \\ 0 & 0 & 0 & -21866 & 1 & 0 & 0 & -10952 \\ 0 & 0 & 0 & -17490 & 0 & 1 & 0 & -2613 \\ 0 & 0 & 0 & -1929 & 0 & 0 & 1 & -683 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ & & & & & & & -73 \end{bmatrix} \quad (16)$$

$$B_c = \begin{bmatrix} -1431 & -5649 \\ -1832 & -2508 \\ -411 & -196 \\ -58 & 0 \\ -10826 & -42514 \\ -13933 & -18475 \\ -03141 & -1429 \\ -450 & 0 \\ 0 & 0 \end{bmatrix} \quad (17)$$

$$C_c = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

$$D_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (19)$$

### 3. THE DIRECTIVE OF FINITE-FREQUENCY IDENTIFICATION

#### 3.1 Area of application

The directive of finite-frequency identification is intended for modeling finite-frequency identification process and correcting parameters of its identification algorithm. The method of finite-frequency identification (Alexandrov, 2005) uses the test signal that is the sum of harmonics whose number does not exceed the order of the state-space model of the plant. The directives differ in the self-tuning of amplitudes and frequencies of the test signal and the duration of identification.

Consider rewriting the plant (1) in form:

$$y = W(s)u + W_f(s)f + \eta; \quad (20)$$

where  $W(s) = C_2(I_n s - A)^{-1}B_2$ ,  
 $W_f(s) = C_2(I_n s - A)^{-1}B_1$

The elements of the transfer matrix  $W(s)$  are

$$w_{pq}(s) = \frac{k_{pq}^{(\gamma_{pq})} s^{\gamma_{pq}} + \dots + k_{pq}^{(1)} s + k_{pq}^{(0)}}{d_{pq}^{(n_{pq})} s^{n_{pq}} + \dots + d_{pq}^{(1)} s + d_{pq}^{(0)}}, \quad (21)$$

$p = \overline{1, r}, q = \overline{1, m};$

where  $\gamma_{pq}$  and  $n_{pq}$  are given numbers.

The following test signal is applied to the plant (20)

$$\begin{aligned} u_q(t) &= \sum_{k=1}^n \rho_{qk} \sin \omega_{qk}(t - t_u), \\ t_u &\leq t < t_u + \tau, q = \overline{1, m} \end{aligned} \quad (22)$$

where  $t_u$  is the time of test signal application,  $\tau$  is the duration of the test signal application (duration of the identification), frequencies  $\omega_{qk}(q = \overline{1, m}, k = \overline{1, n})$  are different positive numbers, amplitudes  $\rho_{qk}(q = \overline{1, m}, k = \overline{1, n})$  are non-negative numbers.

Identification directives are intended for estimation of transfer functions (21). This package is suitable for both discrete and continuous plants. There are two types of identification directives: identification for a plant and identification for a plant in closed loop with the controller. The algorithms of both directive types are similar, but in the case of the added controller, the algorithm is more complicated.

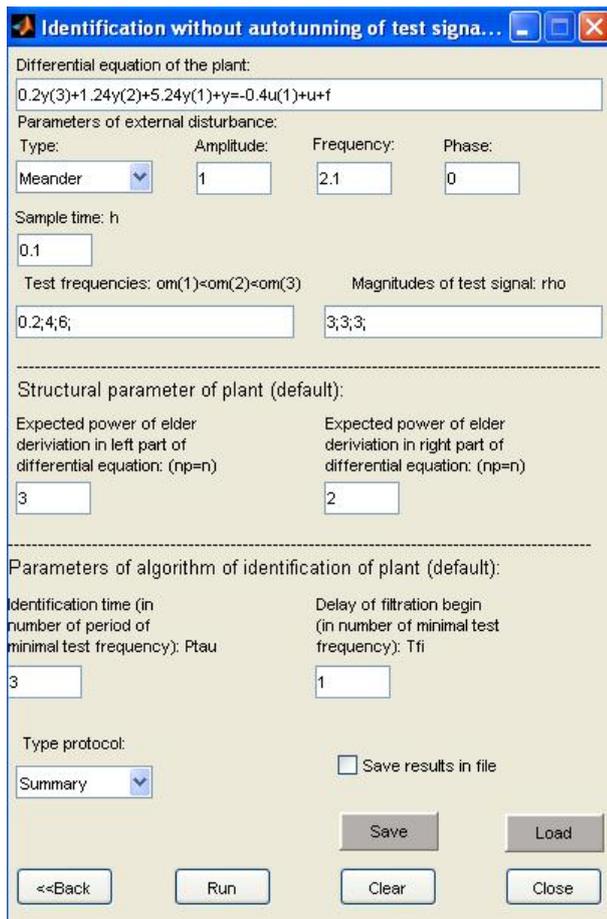


Fig. 3. The interface of the identification directive

The package contains the following identification directives:

- Identification without self-tuning of test signal; In this directive, the duration of identification  $\tau$ , amplitudes  $\rho_{qk}$  and frequencies  $\omega_{qk}$  ( $q = \overline{1, m}, k = \overline{1, n}$ ) are considered given numbers.
- Identification with self-tuning of identification duration; In this directive, the duration of identification  $\tau$  is not given and it is determined during the identification process by using the necessary conditions of identification convergence.
- Identification with self-tuning of test signal amplitudes and identification duration; In this directive, duration of identification and amplitudes  $\rho_{qk}$  ( $q = \overline{1, m}, k = \overline{1, n}$ ) of test signal are not given and are found automatically during the identification process.
- Identification with self-tuning of amplitudes and frequencies of test signal and identification duration; In this directive, frequencies and amplitudes of test signal (22) and duration of identification are not given, but are determined during the identification process.

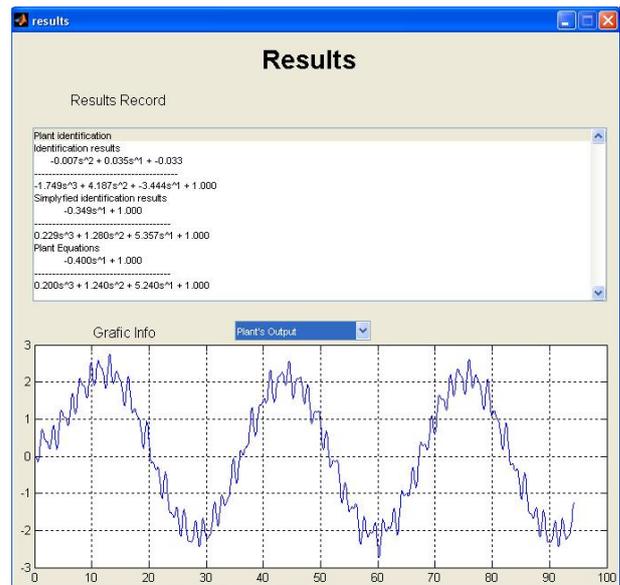


Fig. 4. Identification directive results

### 3.2 Interface

The user interface of the directive for identification without self-tuning of test signal is shown in Fig. 3. The differential equation for plant is

$$0.2y^{(3)} + 1.24y^{(2)} + 5.24y^{(1)} + y = -0.4u + f; \quad (23)$$

For the plant  $y = z$ .

The external disturbance is like the following:

$$f = \text{sign}[\sin(2.1t)]; \quad (24)$$

The vectors of frequencies and amplitudes of the test signal are  $\omega = [0.2, 4, 6]$   $\rho = [3, 3, 3]$ . The test signal itself is:

$$u = 3 \sin(0.2t) + 3 \sin(4t) + 3 \sin(6t); \quad (25)$$

$\gamma_{pq}$  and  $n_{pq}$  in (21) are 2 and 3 accordingly. Parameter identification time determines the duration of identification. Parameter delay during the filtering stage determines the delay of plant transient response.

The results of directive execution are shown in Fig 4. The upper part of the figure displays the directive report, and some graphic information is displayed below it. The identification results are shown in the Table 1.

Table 1. Identification results

	TF nominator	TF denominator
Plant	$-0.4s + 1$	$0.2s^3 + 1.24s^2 + 5.24s + 1$
Result	$-0.349s + 1$	$0.229s^3 + 1.280s^2 + 5.357s + 1$

## 4. DIRECTIVE OF FREQUENCY ADAPTIVE CONTROL

### 4.1 Area of application

The directives for frequency-adaptive control are intended for modeling the adaptation process and correcting parameters of the identification and the adaptation. Directives have the following algorithm: unknown parameters of the plant (1) are determined by using one of the available

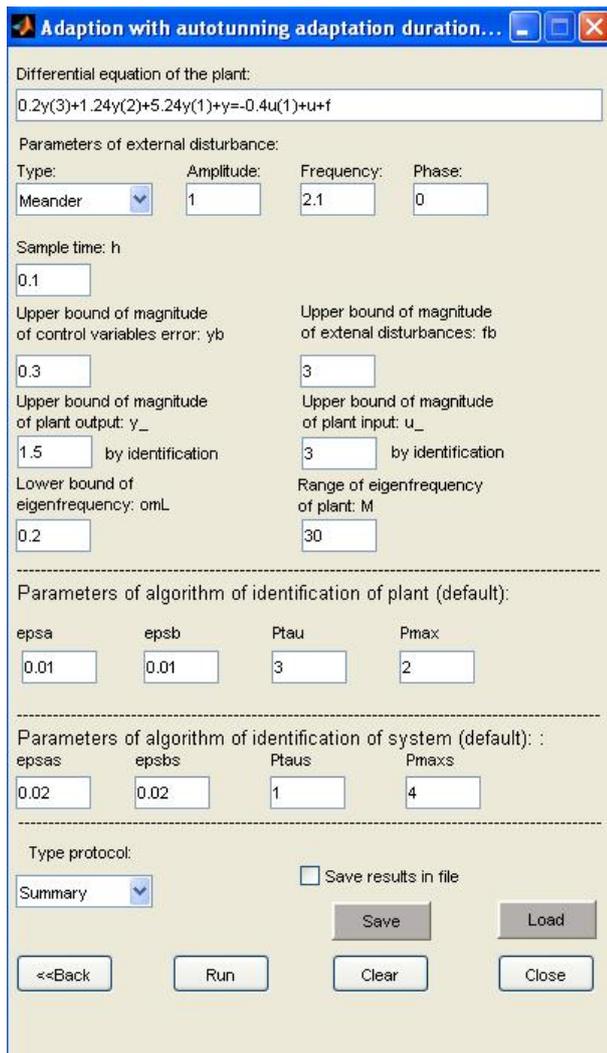


Fig. 5. The user interface for adaptation directive

identification methods. Then controller design directive can be used to develop the corresponding controller for the plant (2). Parameters of the plant slowly change in time, which degrades controller accuracy. New identification is required then. Let the system (1)(2) stay in some stable state. The directive for plant identification in the closed loop with the controller is executed. After the identification new controller is developed. The package contains the following adaptation directives:

- Adaptive control without self-tuning of test signal;
- Adaptive control with self-tuning of identification duration;
- Adaptive control with self-tuning of amplitudes of test signal and duration of identification;
- Adaptive control with self-tuning of amplitudes and frequencies of test signal and duration of identification;

#### 4.2 Interface

The user interface for the directive of adaptive control with self-tuning of test signal amplitudes and duration of identification is shown in Fig. 5. The plant for adaptation is described by equation (23), the external disturbance is

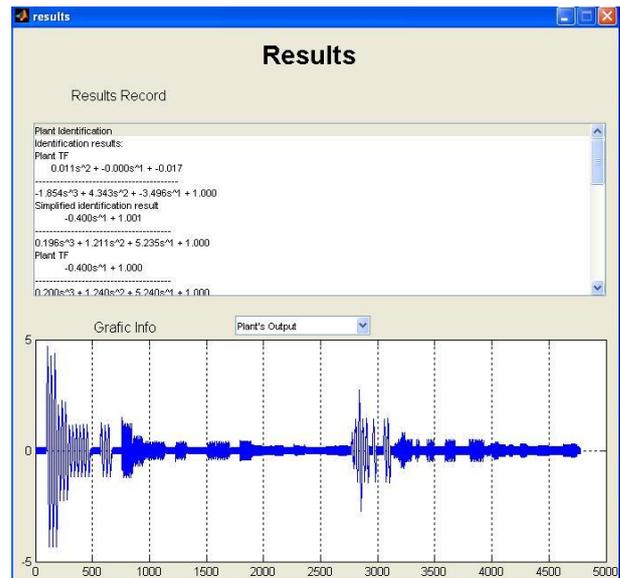


Fig. 6. Adaptive directive results

also similar to the identification example and is described by (24). The sample time is  $h = 0.1$ .

Required steady-state errors of controlled variables and bounds of disturbances are like the following:

$$\begin{aligned} y^* &= 0.3; \\ f^* &= 3; \end{aligned} \quad (26)$$

Upper bounds of input and output signals of the plant during identification are:

$$\begin{aligned} y_- &= 1.5; \\ u_- &= 3; \end{aligned} \quad (27)$$

The range of test frequencies is  $\omega = [0.2; 30.2]$ . This interval is close to the natural frequency band of the plant. Other directive parameters take their default values, but they can be changed when necessary.

The results of the directive execution are shown in Fig 6.

Table 2. Adaptation results

	TF nominator	TF denominator
Plant	$-0.4s + 1$	$0.2s^3 + 1.24s^2 + 5.24s + 1$
Step 1	$-0.4s + 1.001$	$0.196s^3 + 1.211s^2 + 5.235s + 1$
Step 2	$-0.398s + 1$	$0.195s^3 + 1.189s^2 + 5.288s + 1$

The adaptation results are shown in the Table 2. Step 1 shows the identification result of the plant. After this step the following controller was synthesized:

$$w_c(s) = \frac{-3.434s^2 + 4.756s - 1.952}{s^2 - 1.218s + 0.048}; \quad (28)$$

After that a new identification is run. Step 2 in the Table 2 shows the result of this new identification for the plant with controller (28). Then a new controller is synthesized. This controller is described by the equation:

$$w_c(s) = \frac{-2.921s^2 + 4.04s - 1.665}{s^2 - 0.841s + 0.224}; \quad (29)$$

## 5. CONCLUSION

The package "Automatic" gives new possibilities in the field of real-world control systems design. These possibilities are assured by the following factors:

- (1) The package is oriented not only towards theorists, but also towards engineers who actually develop control systems. The package includes directives that solve defined classes of problems, which proves to be very useful in the process of control system development.
- (2) The package gives means to analyze control systems while taking properties of the real world into account, be it structural and parametric uncertainty, unknown disturbances or measurement noises.
- (3) The package directives are intended for solving defined, formalized classes of problems: accuracy control, finite-frequency identification, frequency-adaptive control.

## REFERENCES

- Alexandrov, A.G. and S.Yu. Panin (1997). GAMMA-1PC as CACSD tools for practising engineers. *Proceedings of 7th Symposium on Computer Aided Control System Design (CACSD'97)*, Gent, Belgium, P. 287-292.
- MATLAB User's Guide, MathWorks, 2001
- Alexandrov A.G., Yu.F. Orlov and L.S. Mikhailova (2009). ADAPLAB-3: finite-frequency identification and adaptation toolbox for MATLAB. *Preprints of the 15th IFAC Symposium on System Identification*. Saint-Malo, France, P. 498-503.
- Ljung, L. (1987). *System Identification – Theory for the User*. Prentice-Hall, Englewood Cliffs, New Jersey.
- Alexandrov A.G. (2005). Finite-frequency identification: self-tuning of test signal. *Preprints of the 16th IFAC World Congress*, Prague, CD-ROM.
- Alexandrov A.G., Yu.F. Orlov and L.S. Mikhailova (2012). Package "Automatica" for MATLAB. *Preprints of the 16th IFAC Symposium on System Identification*. Brussels, Belgium. 2012. pp. 1832-1837.