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Package "Automatica" for MATLAB ⋆

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Abstract: The algorithm, the structure and area of application of package "Automatica" are considered. The package is intended for engineers-developers of real-world control system. The purpose of such users is to provide the given tolerance on steady-state error for each controlled variables when the disturbance is unknown but bounded function and the uncertain plant parameters change their values slowly. In this connection, the identification of plant parameters and redesign of controller are required. The package has special structure that is oriented for such users. The basis of this structure is the directives that solve defined class of problems. The package includes three groups of directives: controller design, finite-frequency identification, frequency adaptive control.

Keywords: software, multivariable plant, controller design, identification, adaptive control, unknown-but-bounded disturbance

1. INTRODUCTION

The work quality of real control system is characterized by technical indices: a steady-state error, settling time, maximum overshoot for each controlled variable.

The controller design in terms of these characteristics is complicated because of uncertainty of bounded disturbance and uncertainty of plant coefficients that can change their values. This property of the plant is described by multi-regime plant model.

A package "Automatica" is intended for controller design in such conditions. It is oriented on two user groups of the software. The first group is the researchers who well know the control theory and problems of control of a concrete data domain. The researchers create the program for the solution of real problems (further, we refer such programs as to "directives"). The directive is a program consisting of three parts: a software for user interface, computational part and means for output of intermediate and final results. Each directive solves the defined class of problems of a designing of control algorithm.

The second user group is the engineers-developers of a control system. The purposes of this group and a small time for control system development eliminate a capability of their participation in creation of the software for the solution of their problems. Besides, the necessity of a profound knowledge of the theory of control also handicaps their participation in directives development. An engineer selects the directive which solves his problem from the list of package directives and enters the description of his problem. The solution of the problem is implemented automatically. Analyzing the results, he makes a decision on an acceptability of designed controller.

The package "Automatica" contains three groups of directives: controller design, finite-frequency identification, frequency adaptive control. This package is extension of our early developed software. The directives of the package GAMMA-1PC Alexandrov (1997) for design of controller for MIMO control systems were written in FORTRAN. Now, we developed them in MATLAB. See MATLAB (2001). The package ADAPLAB-3 Alexandrov et al. (2009) is improved through the self-tuning of test signal. It is very important modification. The point is that the basis of adaptive control is the method of finite-frequency identification. This method uses the test signal in which the number of harmonics does not exceed the plant order. The determination of amplitudes and frequencies of test signal is the main complication of this method. The amplitudes are tuned such that the difference between plant output in the presence and in the absence of test signal lies in the allowable bounds. The self-tuning of frequencies allows to decrease the duration of identification. As distinguished from the package ADAPLAB-3 that provides the self-tuning for SISO-plant, package "Automatica" provides self-tuning of test signal for MIMO-plant.

Compare the facilities of our package with MATLAB toolboxes. Control System Toolbox is intended for user of firs group (researchers) who have advanced knowledge of control theory and knowledge about area of application of designed control (ex., aircraft, power engineering and etc.) Such specialists use rich spectrum of m-functions for developing of procedures for solution of problem of

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real world control system design. But this toolbox is not convenient for engineer-developer of control system.

Frequency-domain system identification toolbox is applicable for situation when the disturbance absents or it is white noise (frequency domain method, least-square identification). The method of instrumental variables Ljung (1987) is applicable for arbitrary disturbance but the self-tuning of controlled input is not provided for this method. Thereat, the duration of identification may be too long. Thus, this method is not applicable for adaptive control.

The paper is organized as follows. In section 2, we describe the area of application of package. The section 3,4,5 are devoted to directives for controller design, finite-frequency identification and adaptive control respectively. The area of application, algorithms and the m-functions of each directives are described.

2. AREA OF APPLICATION

Consider asymptotically stable control system described by equations

$$\dot{x} = A^{[re]}x + B_2^{[re]}u + B_1^{[re]}f, y = C_2^{[re]}x + \eta, z = C_1^{[re]}x,$$
$$t_{re-1} \le t < t_{re}, re = 1, 2, \dots, N, \quad (1)$$
$$\dot{x}_c = A_c(t)x_c + B_c(t)y, u = C_c(t)x_c + D_c(t)y, \quad (2)$$

where $[re]$ is the number of regime of plant (1) operation, t_{re} is the moment of termination of re^{th} regime (for simplicity, we assume that values $t_{re}, re = 1, 2, \dots$ are known), $x(t) \in R^n$ is the vector of plant states, $x_c(t) \in R^{n_c}$ is the vector of controller (2) states, $u(t) \in R^m$ is the control, $y(t) \in R^r$ is the measured variables, $z(t) \in R^m$ is the controlled variables, $f(t) \in R^m$ is the unmeasured disturbance, $\eta(t) \in R^r$ is the measurement noise, $A^{(re)}$, $B_1^{(re)}$, $B_2^{(re)}$, $C_1^{(re)}$, $C_2^{(re)}$ are numerical matrices.

Disturbance and noise are bounded polyharmonic functions

$$f_i(t) = \sum_{k=0}^{\infty} f_{ik}\sin(\omega_k t + \varphi_k), \quad \eta_j(t) = \sum_{k=0}^{\infty} \eta_{jk}\sin(\tilde{\omega}_k t + \tilde{\varphi}_k), \quad i = \overline{1,m},\ j = \overline{1,r}, \quad (3)$$

where frequencies ω_k and $\tilde{\omega}_k$ and phases φ_k and $\tilde{\varphi}_k$ ($k = \overline{0,\infty}$) are unknown. The unknown amplitudes f_{ik} and η_{jk} ($i = \overline{1,m}, j = \overline{1,l}, k = \overline{0,\infty}$) meet the conditions

$$\sum_{k=0}^{\infty} |f_{ik}(t)| \le f_i^*,\ \sum_{k=0}^{\infty} |\eta_{jk}(t)| \le \eta_j^*, \quad i = \overline{1,m},\ j = \overline{1,r}, \quad (4)$$

where f_i^* and η_i^* ($i = \overline{1,m}, j = \overline{1,r}$) are known values.

If the conditions (4) are satisfied then the following inequalities hold true

$$|f_i(t)| \le f_i^*,\ |\eta_i(t)| \le \eta_i^*,\ i = \overline{1,m},\ j = \overline{1,r}. \quad (5)$$

Particular case of functions (3) is piece-continues functions which are dissolvable to Fourier series and meet the conditions (4).

Steady-state errors of controlled variables and controls are determined as

$$z_{i,st} = \lim_{t\to\infty} sup|z_i(t)|, \quad i = \overline{1,m}. \quad (6)$$

We assume that the duration $(t_r - t_{r-1})$ of regimes of plant operation are sufficiently long are known with sufficient accuracy.

Control purpose is

$$z_{i,st} \le z_{i,st}^*,\ i = \overline{1,m}, \quad (7)$$

where $z_{i,st}^*(i = \overline{1,m})$ are given numbers.

The control that provides achievement of purpose (7) is referred to as accurate control.

The package is intended for calculation of matrices of controller (2) that provides achievement of control purpose (7) for plant (1) under different levels of uncertainty of plant (1) matrices.

The package contains thee groups of directives: controller design, finite-frequency identification, adaptive control.

Each directive has the graphical interface that provides the input of differential equation of the plant, the disturbances ant their bounds f_i^* ($i = \overline{1,m}$), tolerances z_i^* ($i = \overline{1,m}$) on controlled variables. Tuned parameters of identification and adaptation algorithm have default values but can be determined by user. Besides, a user can select the intermediate results that should be displayed.

Differential equation of the plant are entered in arbitrary form. The most common form is the Lagrange form

$$(\sum_{i=0}^{l_1} L_i^{[re]}s^i)q = (\sum_{i=0}^{l_2} L_i^{u[re]}s^i)u + (\sum_{i=0}^{l_3} L_i^{f[re]}s^i)f,$$
$$y = D^{y[re]}q + \eta, z = D^{z[re]}q, \quad (8)$$

where s is the symbol of Laplace transformation under zero initial conditions, $L_i^{(r)}$, $L_j^{n(r)}$, $L_\mu^{f(r)}$ ($i = \overline{1,l_1}, j = \overline{1,l_2}, \mu = \overline{1,l_0}$), D^y, D^z are numerical matrices, $q(t) \in R^l$ is the vector of intermediate variables.

These equations are transformed to state-space form (1) and directives use them for calculations.

Function `lagrca` executes this transformation.

$$[A_1, B_1, B_2, C_2, C_1] = \texttt{lagrca}(L_1, \cdots, L_{l_1}, L_1^u, \cdots, L_{l_2}^u, L_1^f, \cdots, L_{l_3}^f, D^y, D^z)$$

Each directive contains this function.

3. DIRECTIVES OF CONTROLLER DESIGN

3.1 Area of application

In these directives, plant has single regime of operation. It is described by the equations

$$\dot{x} = Ax + B_2u + B_1f, y = C_2x + \eta, z = C_1x, \quad (9)$$

where matrices are known with high accuracy so the plant (9) is robust stabilizable. It means that there exists the controller matrices

$$\dot{x}_c = A_cx_c + B_cy, u = C_cx_c + D_cy \quad (10)$$

such that, under small deviations of plant (9) matrices, the system (9), (10) is asymptotically stable.

The directives for controller design differ by forms of plant equations.

3.2 Directive 441: Accurate control for plant of first type

Plant of first type is described by equations

$$\dot{x} = Ax + B(u + f), y = x, z = C_1 x. \tag{11}$$

The control purpose is achieved by controller

$$u = Kx \tag{12}$$

that minimize the functional

$$J = \int_0^\infty (z' Q_0 z + u' u) dt, \tag{13}$$

where

$$Q_0 = diag|q_{11}, ...q_{mm}|,\ q_{ii} = \frac{\sum_{k=1}^m f_k^{*2}}{z_{i,st}^{*2}}. \tag{14}$$

The directive use the functions `care` and `lsim`.

```
[K]=care(A,B,Q)
[z]=lsim(A+BK,f)
```

where $Q = C_1' Q_0 C_1$. Here and after, some inputs and outputs in functions descriptions are omitted.

3.3 Directive 442: Accurate control for plant of second type

Plant of second type is described by equations

$$\dot{x} = Ax + B_2 u + B_1 f, y = C_2 x, z = y, \tag{15}$$

besides, it should be minimum-phase.

It means that if we write the plant (15) by equations in input-output form and obtain the equation

$$P(s)y = Q(s)u + Q_1(s)f \tag{16}$$

then the roots of polynomial $det\,[Q(s)]$ have the negative real parts.

Introduce the control

$$\tilde{u} = Q(s)u \tag{17}$$

and transform the equation $P(s)y = \tilde{u}$ to state-space form. Making use the directive 441 and getting vector x of equation (12) through vector y, through equation (11) ($x = F(s)y + \Phi(s)u$), we get the control

$$Q_2(s)u = G(s)y. \tag{18}$$

If the controller (18) is non-realizable then the functional is supplemented by the derivatives of control so as the equation (18) can be transformed to state-space form (10). It leads to sufficient complexity of directives. The description of directives for this case is omitted.

Directives 441 and 442 provides satisfaction of requirements to radius of stability margins

$$r \geq 0.75, \tag{19}$$

where r is the minimal positive number under which the inequalities

$$[I_m + W_c(-j\omega)W(-j\omega)][I_m + W(j\omega)W_c(j\omega)] \geq I_m r^2,$$
$$0 \leq \omega \leq \infty,$$

$$W(s) = C_2(I_n s - A)^{-1} B_2,$$

$$W_c(s) = -C_c(I_{n_c} s - A_c)^{-1} B_c \tag{20}$$

holds true.

The directive contains the functions

```
[P_1,Q,Q_1]=cauio(A,B_2,B_1,C_2)
[F,Phi]=vost(A,B_2,C_2)
[r]=radi(W,W_c)
```

and functions of directive 441.

3.4 Directive 443: Control for plant of common type

Plant of common type is described by the equations (9). This directive is based on the function `hinfric` where matrixes Q_0 are determined by expression (14).

Note that the directives 441 and 442 provide the achievement of control purpose under every given values $z_{i,st}^*$ ($i = \overline{1,m}$). For directive 443, these values should meet the inequality

$$z_{i,st}^* < g_{opt},\ \ i = \overline{1,m}, \tag{21}$$

where g_{opt} is found by function `hinfric`. Besides, the requirements (19) to stability margins of the system can be violated.

4. DIRECTIVE OF FINITE-FREQUENCY IDENTIFICATION

4.1 Area of application

The directive of finite-frequency identification is intended for modeling of finite-frequency identification process and correction of parameters of identification algorithms.

The directive is applicable for identification of real plant in real time.

The method of finite-frequency identification (Alexandrov, 2005) uses the test signal that is the sum of harmonics whose number does not exceed the order of state-space model of the plant. The directives differ by the selftuning of amplitudes, frequencies of test signal and duration of identification.

Consider asymptotically stable plant (9) where $f(t)$ and $\eta(t)$ are unknown but bounded functions and plant coefficients are unknown.

Write the equation (9) as

$$y = W(s)u + W_f(s)f + \eta, \tag{22}$$

where

$$W(s) = C_2(I_n s - A)^{-1} B_2,$$
$$W_f(s) = C_2(I_n s - A)^{-1} B_1.$$

The elements of transfer matrix are

$$W_{pq}(s) = \frac{k_{pq}^{(\gamma_{pq})} s^{\gamma_{pq}} + \cdots + k_{pq}^{(1)} s + k_{pq}^{(0)}}{d_{pq}^{(n_{pq})} s^{n_{pq}} + \cdots + d_{pq}^{(1)} s + d_{pq}^{(0)}}, \quad p = \overline{1,r}, q = \overline{1,m}. \tag{23}$$

For simplicity, we assume the numbers γ_{pq} and n_{pq} are given.

The following test signal is applied to the plant (22)

$$u_q(t) = \sum_{k=1}^n \rho_{qk} \sin \omega_{qk}(t - t_u), \quad t_u \leq t < t_u + \tau,\ \ q = \overline{1,m}, \tag{24}$$

where t_u is the time of test signal application, τ is the duration of test signal application (duration of identification), the frequencies ω_{qk} $(q = \overline{1,m}, k = \overline{1,n})$ are different positive numbers, the amplitudes ρ_{qk} $(q = \overline{1,m}, k = \overline{1,n})$ are non-negative numbers.

Identification is the process of determining the coefficient estimates of the transfer function (23).

4.2 Directive d123mi: identification without self-tuning of test signal

In this directive, the duration of identification τ, amplitudes ρ_{qk} and frequencies ω_{qk} $(q = \overline{1,m}, k = \overline{1,n})$ are given numbers.

The equations (9) with equations (24) are solved by program `lsim`

```
A=A1; B=[B1 B2]; C=C2; D=0;
u1=[u;f;h]; sys=ss(A,B,C,D);
[y]=lsim(sys, u1, tau);
```

The result `y` is applied to the input of Fourier filter whose output gives following frequency domain parameters of the plant

$$\begin{aligned} \alpha_{pqi}(\tau) &= \frac{2}{\rho_{qi}(\tau)} \int_{t_F}^{t_F+\tau} y_p(t) \sin \omega_{qi}(t - t_u) dt, \\ \beta_{pqi}(\tau) &= \frac{2}{\rho_{qi}(\tau)} \int_{t_F}^{t_F+\tau} y_p(t) \cos \omega_{qi}(t - t_u) dt, \\ & p = \overline{1,r}, \quad q = \overline{1,m}, \quad i = \overline{1,n}. \end{aligned} \tag{25}$$

Estimates of the coefficients of the transfer function (23) (at the moment τ) can be found after the system of linear equations

$$\begin{aligned} &\sum_{\mu=0}^{\gamma_{pq}} (j\omega_{qi})^\mu k_{pq}^\mu(\tau) - (\alpha_{pqi}(\tau) + j\beta_{pqi}(\tau)) \times \\ &\times \sum_{\mu=1}^{n_{pq}} (j\omega_{qi})^\mu d_{pq}^\mu(\tau) = \alpha_{pqi}(\tau) + j\beta_{pqi}(\tau) \end{aligned} \tag{26}$$

is solved.

The function `fourmi` gives the estimations of frequency domain parameters (25).

```
[alpha, beta] =
     fourmi(y, u, omega, rho, tau, tu, tF)
```

where `alpha` and `beta` are matrices consisting of components α_{pqi} and β_{pqi}, `omega`, `rho` are vectors consisting of frequencies ω_{qk} and amplitudes ρ_{qk} $(q = \overline{1,m}, k = \overline{1,n}, i = \overline{1,r})$.

Function `freqmi` solves the equation (26) and gives the estimations of plant coefficients

```
[k,d] = freqmi(alpha, beta, omega)
```

where k and d are matrices of coefficients estimations $k_{pq}^{(\mu)}$, $d_{pq}^{(\mu_1)}$ $(p = \overline{1,r}, q = \overline{1,m}, \mu = \overline{0,\gamma^{pq}}, \mu_1 = \overline{1,n^{pq}})$.

4.3 Directive d123dumi: identification with self-tuning of identification duration

In this directive, the duration of identification τ is not given and it is determined during identification process making use the necessary conditions of identification convergence.

The required value τ is

$$\tau = p_\tau T + \psi T, \tag{27}$$

where T is the period of minimal test frequency $\omega_l = min(\omega_{11}, \ldots, \omega_{mn}, \ldots, \omega_{m1}, \ldots, \omega_{mn})$, p_τ and ψ are integer positive numbers. The meaning of p_{tau} is to waiting of the transient process caused by the feed of the test signal.

The following inequalities are checked

$$\begin{aligned} &|\alpha_{pqi}(p_\tau T + \psi T) - \alpha_{pqi}(p_\tau T + (\psi+1)T| \le \\ &\quad \le \varepsilon_{pq}^\alpha \cdot \alpha_{pqi}(p_\tau T + \psi T), \\ &|\beta_{pqi}(p_\tau T + \psi T) - \beta_{pqi}(p_\tau T + (\psi+1)T| \le \\ &\quad \le \varepsilon_{pq}^\beta \cdot \beta_{pqi}(p_\tau T + \psi T), \\ &|d_{pqi}(p_\tau T + \psi T) - d_{pqi}(p_\tau T + (\psi+1)T| \le \\ &\quad \le \varepsilon_{pq}^d \cdot d_{pqi}(p_\tau T + \psi T), \\ &|k_{pqi}(p_\tau T + \psi T) - k_{pqi}(p_\tau T + (\psi+1)T| \le \\ &\quad \le \varepsilon_{pq}^k \cdot k_{pqi}(p_\tau T + \psi T), \end{aligned} \tag{28}$$

where ε_{pq}^α, ε_{pq}^β, ε_{pq}^d, ε_{pq}^k are given sufficiently small numbers referred as to parameters of identification algorithm.

The directive d123mi is used for each values $\psi = 1, 2, \ldots$ until the necessary conditions (28) will be satisfied.

4.4 Directive d123adumi: identification with self-tuning of amplitudes of test signal and duration of identification

In this directive duration of identification and amplitudes ρ_{qk} $(q = \overline{1,m}, k = \overline{1,n})$ of test signal are not given and they are found during the identification process through the inequality

$$\kappa_p \le \kappa_p^*, \tag{29}$$

where

$$\begin{aligned} \kappa_p &= \frac{|y_{pm} - \bar{y}_{pm}|}{|\bar{y}_{pm}|}, \quad p = \overline{1,r}, \\ y_{pm} &= \max_{t_0 \le t \le t_0+\tau} |y_p(t)|, \quad p = \overline{1,r}, \\ \bar{y}_{pm} &= \max_{t_0 \le t \le t_0+\tau} |\bar{y}_p(t)|, \quad p = \overline{1,r}, \end{aligned} \tag{30}$$

where $\bar{y}_p(t)$ is the "natural" output of the plant when the test signal absents $(u(t) = 0)$. Numbers κ_p $(p = \overline{1,r})$ characterizes influence of test signal to the plant output. κ_p^* are the given numbers that characterizes the allowable additions to the "natural" output. Duration of identification increases if κ_p^* decrease.

The self-tuning of amplitudes executes before the start identification process. Thereat, we take into account the limitation of control

$$|u_q(t)| \le u_q^*, \quad q = \overline{1,m}, \tag{31}$$

where u_q^* $(q = \overline{1,m})$ are given numbers.

The self-tuning is executed by function `tunampmi` that includes function `lsim`.

```
[rho]=tunampmi(A, B1, B2, C2, f, nu, uq*,
kappa*, initrho, t)
```

where `initrho` is the initial value of amplitude, `t` is the duration of amplitude self-tuning.

4.5 Directive d123floadumi: identification with self-tuning of amplitudes and frequencies of test signal and duration of identification

In this directive, the frequencies and amplitudes of test signal (24) and duration of identification are not given but determined during identification process.

Test frequencies are chosen from the natural frequencies of the plant. In the contrary case, identification time τ may be unallowable long. This interval is determined by time constant of the plant transfer functions.

Write transfer functions as

$$w_{pq}(s) = k_{pq}^{(0)} \times$$
$$\times \frac{\prod_{k=1}^{\check{c}_{pq}} \left(\check{T}_{pqk}s+1\right) \prod_{k=1}^{\breve{c}_{pq}} \left(\breve{T}_{pqk}^2 s^2+2\breve{T}_{pqk}\breve{\xi}_{pqk}s+1\right)}{\prod_{k=1}^{\bar{c}_{pq}} \left(\bar{T}_{pqk}s+1\right) \prod_{k=1}^{\tilde{c}_{pq}} \left(\tilde{T}_{pqk}^2 s^2+2\tilde{T}_{pqk}\tilde{\xi}_{pqk}s+1\right)},$$
$$p = \overline{1,m}, \quad q = \overline{1,r}. \tag{32}$$

Denote

$$\omega_{lpq} = \min_{k,p}\{\frac{1}{\check{T}_{pqk}}, \frac{1}{\breve{T}_{pqk}}, \frac{1}{\bar{T}_{pqk}}, \frac{1}{\tilde{T}_{pqk}}\},$$
$$\omega_{upq} = \max_{k,p}\{\frac{1}{\check{T}_{pqk}}, \frac{1}{\breve{T}_{pqk}}, \frac{1}{\bar{T}_{pqk}}, \frac{1}{\tilde{T}_{pqk}}\}. \quad q = \overline{1,m} \tag{33}$$

Test frequencies applied to the p^{th} input of the plant should meet the inequality

$$\omega_{lpq} \leq \omega_{qk} \leq \omega_{upq} (q = \overline{1,m}, k = \overline{1,n}). \tag{34}$$

Low bounds are determined making use of procedure P1:

(1) Plant is excited by controls
$$u_q(t) = \rho_{q1} \sin \omega_q^{in} t, \quad q = \overline{1,m}, \tag{35}$$
where ω_q^{in} $(q = \overline{1,m})$ are given numbers (minimal values of estimations of low bounds), amplitudes ρ_{q1} are calculated by function `tunampmi`;

(2) The estimations of low bounds are calculated as
$$\omega_{lpq}^{(1)} = \frac{\omega_q^{in} \alpha_{pq}(\tau)}{\beta_{pq}(\tau)}, \; p = \overline{1,r}, \; q = \overline{1,m}, \tag{36}$$
where τ are determined by function `tunFourmi` .

(3) Repeat step (1)-(2) for $\omega = \omega_q^{in}/2$ and find new estimations ω_{lpq}^2 and so on, until the following condition will be satisfied
$$\frac{|\omega_{lpq}^{(i)} - \omega_{lpq}^{(i-1)}|}{\omega_{lpq}^{(i)}} \leq \varepsilon_{pq}^{om}, \; i = 2, \ldots, maxom, \tag{37}$$
where ε_{pq}^{om} are given numbers.

Low bound are calculated by function `tunfLomi`. It includes functions `lsim`, `fourmi` (with the search of τ), `tunampmi`.

```
[omega_low]=tunfLomi(A, B1, B2, C2, f, nu,
uq*, kappap*, initrho,w_in, eps_omt_pq1)
```

where `omega_low` is the matrix of estimation of low bound of natural frequencies.

The estimation of high bound is determined as

$$\omega_{upq} = M_{pq} \omega_{lpq}, \tag{38}$$

where M_{pq} are given numbers that are estimations of intervals of natural frequencies of each transfer function.

5. DIRECTIVE OF FREQUENCY ADAPTIVE CONTROL

Directive of frequency adaptive control is intended for modeling of adaptation process and correction of parameters of identification and adaptation. Directives is applicable for adaptation of multi-regime plant (1).

5.1 Directive of adaptation for single-regime plant

This directive is the simple union of identification directives and directives of accurate control. Twelve directives were developed by this way. The names of these directives are the union of corresponding names of identification directives and directives of controller design. For example, `d123sumu442` is the directive of adaptive control with identification of plant of second type without self-tuning of test signal.

5.2 Directives of adaptation for multi-regime plant. Directive d323mi: adaptive control without self-tuning of test signal

Let us develop the controller (10) making use one of adaptation directives without self-tuning of test signal for the first regime. At the moment t_1, the plant (1) coefficients changed. Let the system (1)(10) keeps stable (we assume this property is true for following adjoining regimes, the controller developed for i^{th} regime provides stability of plant in $(i+1)^{th}$) regime.

For identification of plant in the second regime, the following test signal with given parameters is applied to the plant

$$v_l^{[2]}(t) = \sum_{k=1}^{n} \rho_{lk}^{[2]} \sin \omega_{lk}^{[2]} (t - t_n^{[2]}),$$
$$t_n^{[2]} \leq t \leq t_n^{[2]} + \tau^{[2]}, \quad l = \overline{1,m}, \tag{39}$$

where index "[2]" means the second regime and plant (1) is described by equations

$$\dot{x} = A^{[2]}x + B_1^{[2]}f + B_2^{[2]}(u + v^{[2]}),$$
$$y = C_2^{[2]}x + \eta, z = C_1^{[2]}x. \tag{40}$$

The outputs of plant and controller are applied to the Fourier filter and, making use of functions `fourmi`, we found

$$\alpha^y_{pli}(\tau^{[2]}) = \frac{2}{\rho_{li}(\tau^{[2]})}\int_{t_F^{[2]}}^{t_F^{[2]}+\tau^{[2]}} y_p(t)\sin\omega^{[2]}_{li}(t-t_u)dt,$$
$$\beta^y_{pli}(\tau^{[2]}) = \frac{2}{\rho_{li}(\tau^{[2]})}\int_{t_F^{[2]}}^{t_F^{[2]}+\tau^{[2]}} y_p(t)\cos\omega^{[2]}_{li}(t-t_u)dt,$$
$$\alpha^u_{qli}(\tau^{[2]}) = \frac{2}{\rho_{li}(\tau^{[2]})}\int_{t_F^{[2]}}^{t_F^{[2]}+\tau^{[2]}} u_q(t)\sin\omega^{[2]}_{li}(t-t_u)dt,$$
$$\beta^u_{qli}(\tau^{[2]}) = \frac{2}{\rho_{li}(\tau^{[2]})}\int_{t_F^{[2]}}^{t_F^{[2]}+\tau^{[2]}} u_q(t)\cos\omega^{[2]}_{li}(t-t_u)dt,$$
$$p=\overline{1,r},\quad q=\overline{1,m},\quad l=\overline{1,m},\quad i=\overline{1,n}. \tag{41}$$

Then, we calculate frequency parameters by the formulaes

$$\alpha_{pqi}(\tau^{[2]}) = \frac{\alpha^y_{pli}(\tau^{[2]})\alpha^u_{qli}(\tau^{[2]}) + \beta^y_{pli}(\tau^{[2]}\beta^u_{qli}(\tau^{[2]}))}{\alpha^{u2}_{qli}(\tau^{[2]}) + \beta^{u2}_{qli}(\tau^{[2]})},$$
$$\beta_{pqi}(\tau^{[2]}) = \frac{-\alpha^u_{qli}(\tau^{[2]})\beta^u_{qli}(\tau^{[2]}) + \beta^y_{pli}(\tau^{[2]})\alpha^u_{qli}(\tau^{[2]})}{\alpha^{u2}_{qli}(\tau^{[2]}) + \beta^{u2}_{qli}(\tau^{[2]})}. \tag{42}$$

Making use of function `freqmi` that calculates formulaes (42), we find estimations of coefficients of plant (40) transfer matrices.

Amplitudes and frequencies of test signals (39) may be frequently calculated making use of results of plant identification in the first regime and corresponding controller because of the plant coefficients in i^{th} and $(i+1)^{th}$ regimes are sufficiently close. If they are not close then the directives d323dumi, d323adumi, d323floadumi are applicable for such case. In these directives, the identification by directives d123dumi, d123adumi, d123floadumi are used on the second and following regimes.

6. EXAMPLE

Let's consider an example of using `d123mi441` directive *(adaptive control without self-tuning of test signal)*.

The plant has follow form

$$y(t) = W(p)u(t) + W_f(p)f(t),$$

where

$$W(p) = \begin{pmatrix} \dfrac{2s+5}{s^3+6.2s^2+26.2s+5} & \dfrac{0.75}{s+0.25} \\ \dfrac{40}{s+10} & \dfrac{2}{s^2+1.33s+0.33} \end{pmatrix},$$
$$W_f(p) = \begin{pmatrix} \dfrac{2s+5}{s^3+6.2s^2+26.2s+5} \\ \dfrac{2}{s^2+1.33s+0.33} \end{pmatrix}.$$

External disturbances are $f_1(t) = f_2(t) = \sin(0.5t)$.

Test signals chosen as follows

$$u_1(t) = 0.1\sin(0.2t) + 0.5\sin(4t) + 0.5\sin(8t),$$
$$u_2(t) = 0.1\sin(0.1t) + 0.5\sin(3t) + 0.5\sin(7t).$$

Results of identification of the plant transfer matrix is

$$\hat{W}(p) = \begin{pmatrix} \dfrac{1.99s+5.0}{s^3+6.19s^2+26.2s+5.0} & \dfrac{0.75}{s+0.25} \\ \dfrac{39.0}{s+9.6} & \dfrac{1.99}{s^2+1.33s+0.33} \end{pmatrix}.$$

The directive also provides a graphical result, that shown at fig. 1. The figure shows the adaptation stage of the controller in the open loop for both controlled variables (stage of the identification of model parameters of plant) and after adaptation stage, where plant closed by controller.

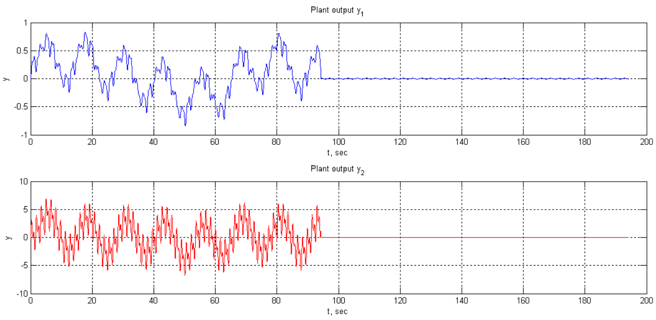

Fig. 1. Results of the directive d123mi441

7. CONCLUSIONS

The package "Automatica" gives new possibilities for design of real-world control system. These possibilities are provided by following:

1. The package is intended for engineers-developers of control system. It is provided by structure of package. The package includes the directives that solve defined class of problems of engineers-developers.
2. The package takes into account the properties of real world such as the accuracy of control, the uncertainty of plant parameters and the multi-regime plants, the uncertainty of disturbances.
3. The algorithms of directives are based on the new results of control theory and therefore the package is a bridge between the theory and practice.

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