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FINITE–FREQUENCY IDENTIFICATION: MODEL VALIDATION AND BOUNDED TEST SIGNAL

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Abstract: Frequency method of identification for a linear continuous plant in the presence of unknown-but-bounded disturbance is developed.
A way of a model validation is proposed. It is based upon enclosing of a plant by a feedback that is designed using the plant coefficients estimates.
The harmonic test signal for unstable plant has a growing peak-to-peak value. The algorithm of the test signal amplitude tuning is designed for the cases of the bounded input of plant. It uses the feedback too.

Keywords: Identification, tests, frequency responses, bounded disturbances, validation.

1. INTRODUCTION

In recent years several methods of linear plant identification in the presence of an unknown-but-bounded disturbance such as the least square techniques (Wahlberg and Ljung, 1992; Milanese, 1994) and the recurrent targetal inequalities method (Fomin, *et al.* 1981; Yakubovich, 1988) have been developed.

The classical frequency approach (Eykhoff, 1974; Unbehauen and Rao, 1990) where a plant is excited by a harmonic test signal, allows to identify a stable plant if a bounded disturbance does not contain the test signal harmonics. This approach for an unstable plant and disturbance of a general view has been developed in the papers (Alexandrov, 1993, 1994) where it was named a finite-frequency identification. The last method uses a harmonic test signal with a growing peak-to-peak value and that is why it is necessary to change identification algorithm for a bounded input of a plant.

In this paper two problems are considered: validation (Ljung, 1987) of the plant model obtained as a result of finite-frequency identification and the way of finite-frequency identification under a bounded test signal. They are solved through the use of the feedback to form the test signal.

2. FINITE–FREQUENCY IDENTIFICATION

2.1 Plant model.

Consider a completely controllable plant described by the differential equation

$$y^{(n)} + d_{n-1}y^{(n-1)} + \cdots + d_1\dot{y} + d_0 y = k_\gamma u^{(\gamma)} + \cdots + k_0 u + f, \quad t \geq t_0 \qquad (1)$$

where $y(t)$ is a measured output, $u(t)$ is an input to be controlled, $f(t)$ is an unknown-but-bounded disturbance, $y^{(i)}$, $u^{(j)}$, $(i=\overline{1,n},\ j=\overline{1,\gamma})$ are derivatives of these functions. The coefficients d_i, k_j, $(i=\overline{0,n-1},\ j=\overline{0,\gamma})$ are unknown numbers, n is known, $\gamma < n$; the initial conditions $y^{(i)}(t_0)$ and disturbance $f(t)$ satisfy the following inequalities

$$|y^{(i)}(t_0)| \le \varepsilon_0, \quad i=\overline{1,n-1}, \quad |f(t)| \le f^* \tag{2}$$

where ε_0 and f^* are positive numbers.

The plant input that is referred to as a test signal is

$$u(t) = \rho e^{\lambda(t-t_0)} \sum_{k=1}^{n} \sin\omega_k(t-t_0) \tag{3}$$

where ρ, λ, ω_k $(k=\overline{1,n})$ are specified positive numbers, ρ is an amplitude of the test signal, ω_k $(k=\overline{1,n})$ are the test frequencies $\omega_i \ne \omega_j$, $i \ne j$ $(i,j=\overline{1,n})$.

Identification problem is to determine the plant coefficients estimates $\hat{d}_i$, $\hat{k}_j$ $(i=\overline{0,n-1})$ $(j=\overline{0,\gamma})$ such that the identification errors

$$\Delta d_i = d_i - \hat{d}_i, \quad \Delta k_j = k_j - \hat{k}_j$$

meet the demands

$$|\Delta d_i| \le \varepsilon_i^d, \ |\Delta k_j| \le \varepsilon_j^k \quad (i=\overline{0,n-1},\ j=\overline{0,\gamma}) \tag{4}$$

where ε_i^d and ε_j^k $(i=\overline{0,n-1},\ j=\overline{0,\gamma})$ are specified numbers.

2.2 Frequency domain parameters (FDP).

Let C_0 denote an estimate of an unstability degree of the plant (1) s^*. $C_0 > s^* = \max\{\mathrm{Re}\,s_1,\ldots,\mathrm{Re}\,s_n\}$, where s_i $(i=\overline{1,n})$ are roots of the polynomial $d(s) = s^n + d_{n-1}s^{n-1} + \cdots + d_0$. For a stable plant $C_0 = 0$. C_0 may be obtained by experiment (Alexandrov, 1992) and that is why it is assumed to be known.

If the plant output $y(t)$ is multiplied by $e^{-\lambda(t-t_0)}$ and the product is applied to Fourier's filter input then the filter outputs give the frequency domain parameters estimates

$$\begin{aligned}
\hat{\alpha}_k(\delta) &= \tfrac{2}{\rho\tau}\int_{t_F}^{t_F+\tau} y(t)e^{-\lambda(t-t_0)}\sin\omega_k(t-t_0)dt = \\
&= \alpha_k + e_k^{\alpha}(\delta), \quad k=\overline{1,n}, \\
\hat{\beta}_k(\delta) &= \tfrac{2}{\rho\tau}\int_{t_F}^{t_F+\tau} y(t)e^{-\lambda(t-t_0)}\cos\omega_k(t-t_0)dt = \\
&= \beta_k + e_k^{\beta}(\delta), \quad k=\overline{1,n}.
\end{aligned} \qquad \lambda > C_0, \tag{5}$$

where t_F is a filtering start time $(t_F \ge t_0)$, τ is a filtering time, $\tau_d = t_F - t_0$ is a delay of a filtering start, $\delta = \tau_d + \tau$ is an identification time, $\lambda > C_0$, $\varepsilon_k^{\alpha}(\delta)$ and $\varepsilon_k^{\beta}(\delta)$ $(k=\overline{1,n})$ *are the filtering errors*, α_k and β_k $(k=\overline{1,n})$ *are frequency domain parameters (FDP)* of a plant. The FDP are linked (Alexandrov, 1989) with the plant transfer function $w(s) = k(s)/d(s)$ by the expressions:

$$\alpha_k = \mathrm{Re}\,w(\lambda+j\omega_k), \ \beta_k = \mathrm{Im}\,w(\lambda+j\omega_k), \ k=\overline{1,n}. \tag{6}$$

If the test frequencies ω_k $(k=\overline{1,n})$ are multiple of some basic frequency ω^* (that is $\omega_k = L_k\omega^*$ where L_k are positive integer numbers $(k=\overline{1,n})$), τ_d and τ are multiple of the reference period $T^* = \frac{2\pi}{\omega^*}$, then the filtering errors satisfy (Alexandrov, 1994) the inequalities:

$$|e_k^{\alpha}(\delta)| \le v(\delta), \quad |e_k^{\beta}(\delta)| \le v(\delta) \quad k=\overline{1,n} \tag{7}$$

where

$$\begin{aligned}
v(\delta) &= \frac{2q_1 f^*}{\rho\tau s^{**}}\left[\frac{1-e^{-q\tau}}{qe^{q\tau_d}} - \frac{1-e^{-\lambda\tau}}{\lambda e^{\lambda\tau_d}}\right] + \\
&+ \frac{2q_2}{\rho\tau q e^{q\tau_d}}\left(1-e^{-q\tau}\right) + \frac{2q_3}{\tau q e^{q\tau_d}}\left(1-e^{-q\tau}\right),
\end{aligned} \tag{8}$$

$s^{**} = s^* + \varepsilon_1$, $q = -s^{**} + \lambda$ $(q,\lambda > 0)$, ε_1 is a sufficiently small positive number, q_1 and q_3 are some positive numbers, q_2 is a function of initial conditions and value $f(t_0)$.

From the expressions (7) and (8) it is easily obtained that the filtering errors are the vanishing functions for any bounded disturbance:

$$\lim_{\delta\to\infty} e_k^{\alpha}(\delta) = \lim_{\delta\to\infty} e_k^{\beta}(\delta) = 0. \tag{9}$$

2.3 Frequency equations.

These equations have the view

$$\begin{aligned}
&\hat{k}(\lambda+j\omega_k) - (\hat{\alpha}_k + j\hat{\beta}_k)\hat{d}(\lambda+j\omega_k) = \\
&= (\hat{\alpha}_k + j\hat{\beta}_k)(\lambda+j\omega_k)^n, \quad k=\overline{1,n}
\end{aligned} \tag{10}$$

where $\hat{k}(s) = \hat{k}_\gamma s^\gamma + \cdots + \hat{k}_0$, $\hat{d}(s) = \hat{d}_{n-1}s^{n-1} + \cdots + \hat{d}_1 s + \hat{d}_0$ are the searched polynomials of plant model. (Here and further the argument δ in the notations of the FDP estimates and solutions $\hat{d}_i(\delta), \hat{k}_j(\delta)$ $(i=\overline{0,n-1},\ j=\overline{0,\gamma})$ is often omitted).

The equations (10) under $\hat{\alpha}_k = \alpha_k$, $\hat{\beta}_k = \beta_k$ $(k=\overline{1,n})$ follow from Bezout-Identity

$$d(s)\hat{k}(s) - k(s)\hat{d}(s) = k(s)s^n \tag{11}$$

after dividing the identity by the polynomial $d(s)$ and a substitution $s = s_k = \lambda + j\omega_k$ $\left(k = \overline{1,n}\right)$. The identity (11) has a unique solution

$$\tilde{\hat{d}}(s) = d(s) - s^n, \quad \hat{k}(s) = k(s), \qquad (12)$$

which is the unique solution of the frequency equations (10) under $\hat{\alpha}_k = \alpha_k$, $\hat{\beta}_k = \beta_k$ $\left(k = \overline{1,n}\right)$ and any positive λ and w_k $\left(k = \overline{1,n}\right)$ (Alexandrov, 1989).

Let δ^* denote an identification time such that the requirements on identification accuracy (4) are fulfilled. The time δ^* always exists. This follows from the property (9) and continuous dependence of the frequency equations solution on small variations of their coefficients and right parts.

It is further assumed that identification time is given a priory, however this value (denoted by $\bar{\delta}$) is examined by the necessary conditions of identification convergence:

$$|\hat{d}_i(\delta) - \hat{d}_i(\delta - T^*)| \le \varepsilon_i^d, \quad |\hat{k}_j(\delta) - \hat{k}_j(\delta - T^*)| \le \varepsilon_j^k \quad (13)$$

$\left(i = \overline{0, n-1}\right)$, $\left(j = \overline{0,\gamma}\right)$, under $\delta = \bar{\delta}$ and

$$\max\{\bar{s}_1, \ldots, \bar{s}_n\} < C_0, \qquad (14)$$

where $\bar{s}_i$ $\left(i = \overline{1,n}\right)$ are roots of polynomial $\hat{d}(s)$.

Algorithm 2.1 (the finite-frequency identification): measure Fourier's filter outputs for the time moments $\bar{\delta}$ (it is multiple T^*) and $\bar{\delta} + T^*$, solve the frequency equations (10) for these moments and examine the necessary conditions (13), (14).

3. PROBLEM STATEMENT

Let the estimates $\hat{d}_i(\bar{\delta})$, $\hat{k}_j(\bar{\delta})$ $\left(i = \overline{0, n-1},\ j = \overline{0,\gamma}\right)$ be found on the base of algorithm 2.1. The prescribed time $\bar{\delta}$ may occur less than δ^*.

Problem A (model validation). Find a form of the test signal $u(t)$ dependent on the plant coefficients estimates and allowing to examine the achievement of the identification purpose (4) experimentally.

The second problem arises because of the plant input restriction:

$$|u(t)| \le u^* \qquad (15)$$

where u^* is a given number.

In this case the identification time is limited by a value δ_b when the test signal (3) reaches the boundary u^*. From (3) and (15) it follows that

$$\delta_b \le \frac{1}{\lambda} \ln \frac{u^*}{\rho n.} \qquad (16)$$

If the amplitude ρ is decreased to increase δ_b then the time δ^* may increase. Moreover, it may occur that the amplitude ρ under which the inequalities (4) are fulfilled (such amplitude is denoted by ρ^*) does not exist.

Problem B (identification under the bounded test signal). Find the existence conditions of the value ρ^* and design the algorithm of this signal amplitude tuning to ρ^*.

4. MODEL VALIDATION

4.1 Approach essence.

Form the searched test signal as a solution of the following differential equation

$$g_{n-1}u^{(n-1)} + \cdots + g_0 u = r_{n-1}y^{(n-1)} + \cdots$$
$$+ r_0 y + l_{n-2}\tilde{u}^{(n-2)} + \cdots + l_0 \tilde{u}. \qquad (17)$$

Here l_i $\left(i = \overline{0, n-2}\right)$ are given numbers; g_i, r_i $\left(i = \overline{0, n-1}\right)$ are numbers determined from the identity

$$\hat{d}(s)g(s) - \hat{k}(s)r(s) = \psi(s), \qquad (18)$$

where $\psi(s)$ is a given polynomial of the degree $2n-1$; $\hat{d}(s) = s^n + \tilde{\hat{d}}(s)$.

The test signal is

$$\tilde{u}(t) = \tilde{\rho} e^{\tilde{\lambda}(t - \tilde{t}_0)} \sum_{k=1}^{n} \sin \tilde{\omega}_k (t - \tilde{t}_0), \quad \tilde{t}_0 > t_0 + \delta \quad (19)$$

where $\tilde{\rho}$, $\tilde{\lambda}$, $\tilde{\omega}_k$ $\left(k = \overline{1,n}\right)$ are given positive numbers that may coincide with the corresponding parameters of the signal (3), $\tilde{\lambda} > \tilde{C}_0$, where $\tilde{C}_0$ is an estimate of an unstability degree $\tilde{s}^*$ of the system (1), (17), $\tilde{C}_0 > \tilde{s}^*$. $s^* = \max\{\tilde{s}_1, \cdots, \tilde{s}_{2n-1}\}$, $\tilde{s}_i$ $\left(i = \overline{1, 2n-1}\right)$ are roots of the following characteristic polynomial of this system

$$\varphi(s) = d(s)g(s) - k(s)r(s). \qquad (20)$$

The system (1), (17) may be written as "a plant"

$$\varphi(s)u = l(s)d(s)\tilde{u} + m(s)r(s)f. \qquad (21)$$

The transfer function "the plant" (21) is

$$w_u(s) = l(s)d(s)/\varphi(s) \qquad (22)$$

It is obvious that if polynomials $\hat{d}(s)$ and $\hat{k}(s)$ tend to $d(s)$ and $k(s)$ respectively then $\varphi(s)$ and $w_u(s)$

tend to $\psi(s)$ and $\hat{d}(s)l(s)/\psi(s)$. Therefore a plant model is validated if the differences of the values $w_u(\tilde{s}_k)$ $(k=\overline{1,n})$ determined experimentally and the values

$$w_u^*(\tilde{s}_k) = \hat{d}(\tilde{s}_k)l(\tilde{s}_k)/\psi(\tilde{s}_k) \quad k=\overline{1,n}, \qquad (23)$$

to be calculated are sufficiently small.

4.2 Frequency domain parameters of feedback output.

Definition 4.1 A set of 2n numbers

$$\theta_k = \mathrm{Re}\, w_u(\tilde{s}_k), \quad \gamma_k = \mathrm{Im}\, w_u(\tilde{s}_k), \quad k=\overline{1,n} \qquad (24)$$

is called the frequency domain parameters of feedback output.

The FDP of plant (1) may be calculated using the numbers (24). In fact, the expressions (20) and (22) give $w_u(s) = \frac{w_l(s)}{1-w(s)w_c(s)}$, where $w_c(s) = \frac{r(s)}{g(s)}$, $w_l(s) = \frac{l(s)}{g(s)}$ and therefore

$$\tilde{\alpha}_k = \mathrm{Re}\, w(\tilde{s}_k), \quad \tilde{\beta}_k = \mathrm{Im}\, w(\tilde{s}_k), \quad k=\overline{1,n}, \qquad (25)$$

where

$$w(\tilde{s}_k) = \frac{(\theta_k + j\gamma_k) - w_l(\tilde{s}_k)}{w_c(\tilde{s}_k)(\theta_k + j\gamma_k)}. \qquad (26)$$

The FDP estimates $\hat{\theta}_k(\tilde{\delta})$ and $\hat{\gamma}_k(\tilde{\delta})$, $(k=\overline{1,n})$, are found as Fourier's filter for "the plant" (21): in the expressions (5) $y(t)$, ρ, ω_k, λ, δ, are replaced by $u(t)$, $\tilde{\rho}$, $\tilde{\omega}_k$, $\tilde{\lambda}$, $\tilde{\delta}$ correspondly, where $\tilde{\delta}$ is a validation time. Using these estimates the plant coefficients estimates $\hat{d}_i(\tilde{\delta})$ and $\hat{k}_j(\tilde{\delta})$ $(i=\overline{0,n-1},\ j=\overline{0,\gamma})$ are determined by way of solution of the frequency equations (10) after the substitution of $\hat{\alpha}_k$, $\hat{\beta}_k$, λ and ω_k by $\hat{\tilde{\alpha}}_k$, $\hat{\tilde{\beta}}_k$, $\tilde{\lambda}$ and $\tilde{\omega}_k$ $(k=\overline{1,n})$, where $\hat{\tilde{\alpha}}_k$ and $\hat{\tilde{\beta}}_k$ are calculated by formulae (25) (26) in which the FDP of feedback θ_k and γ_k $(k=\overline{1,n})$ are replaced by their estimates.

Assertion 4.1 Identification errors satisfy the equations:

$$\sum_{j=0}^{\gamma} \Delta k_j \tilde{s}_k^j - w(\tilde{s}_k)\sum_{i=0}^{n-1} \Delta d_i \tilde{s}_k^i =$$

$$= \nu(\tilde{s}_k)\left[(\theta-\theta_k^*) + j(\gamma-\gamma_k^*)\right] \ (k=\overline{1,n}) \qquad (27)$$

where $\nu(\tilde{s}_k) = \psi(\tilde{s}_k)\varphi(\tilde{s}_k)/d(\tilde{s}_k)l(\tilde{s}_k)r(\tilde{s}_k)$, $\theta_k^* = \mathrm{Re}\, w_u^*(\tilde{s}_k)$, $\gamma_k^* = \mathrm{Im}\, w_u^*(\tilde{s}_k)$, $(k=\overline{1,n})$.

Assertion proof is given in Appendix.

The left parts of the equations (27) and (10) coincide (accurate up to the notations) and therefore the equations (27) have unique solution for identification errors. It means there exists the tolerances ε_k^θ and ε_k^γ $(k=\overline{1,n})$ such that under conditions

$$|\theta_k - \theta_k^*| \le \varepsilon_k^\theta, \quad |\gamma_k - \gamma_k^*| \le \varepsilon_k^\gamma, \quad k=\overline{1,n}, \qquad (28)$$

identification purpose (4) is achievable.

4.3 Validation algorithm.

The inequalities (28), which is the sufficient conditions of achieving of the identification purpose, contain the FDP of feedback but as a matter of fact it is their estimates $\hat{\theta}_k(\tilde{\delta})$, $\hat{\gamma}_k(\tilde{\delta})$ $(k=\overline{1,n})$ that are only known. That is why the inequalities (28) are replaced by the following conditions

$$|\hat{\theta}_k(\tilde{\delta}) - \theta_k^*| \le \varepsilon_k^\theta, \quad |\hat{\gamma}_k(\tilde{\delta}) - \gamma_k^*| \le \varepsilon_k^\gamma, \quad k=\overline{1,n}. \qquad (29)$$

A distinction feature of conditions (29) in comparing with (28) consists in the existence of such correlation of the filtering errors $e_k^\alpha(\delta)$, $e_k^\beta(\delta)$ and $e_k^\theta(\tilde{\delta})$, $e_k^\gamma(\tilde{\delta})$ under that the demands (4) are not met but the inequalities (29) is fulfilled. It may be occurred under a small time $\tilde{\delta}$.

Algorithm 4.1 (model validation): measure Fourier's filter outputs $\hat{\theta}_k$ and $\hat{\gamma}_k$ at time moments $\bar{\tilde{\delta}}$ and $\bar{\tilde{\delta}} + T^*$, check the following conditions that for the *model validation* must be fulfilled: *(i)* the necessary conditions (13), (14), for the estimates $\hat{d}_i(\tilde{\delta})$ and $\hat{k}_j(\tilde{\delta})$ $(i=\overline{0,n-1},\ j=\overline{0,\gamma})$; *(ii)* the conditions (29) under $\tilde{\delta} = \bar{\tilde{\delta}}$; *(iii)* the conditions (29) under $\tilde{\delta} = \bar{\tilde{\delta}} + \gamma\bar{\tilde{\delta}}$, $\gamma > 1$.

If *(i)* is violated then go to the beginning of algorithm increasing $\bar{\tilde{\delta}}$. If *(ii)* and *(iii)* are violated or while fulfilling *(ii)* the conditions *(iii)* are violated then *the plant model is not validated.*

5. IDENTIFICATION UNDER THE BOUNDED TEST SIGNAL

It may be extracted one important case when amplitude ρ^* in problem B is calculated a priori. If the plant (1) is stable (the roots of $d(s)$ lie the left half-plane) and disturbance $f(t)$ does not contain the test frequencies then the filtering errors tend to zero under $\lambda = 0$. This follows from the proof of the expression (8). Under $\lambda = 0$ the searched amplitude $\rho^* \le u^*/n$.

Now consider a general case.

Assertion 5.1 There always exists an amplitude ρ^* and boundaries f^{**} and ε_0^* of the disturbance and the initial conditions so that the identification purpose (4) is achievable under restriction (15) on the test signal.

Assertion proof is given in Appendix.

If the conditions hold

$$f^* \leq f^{**} \quad \text{and} \quad \varepsilon_0 < \varepsilon_0^*. \tag{30}$$

the following converged algorithm solves the identification problem under an bounded test signal (for simplicity, the identification time δ^* is determined from the necessary conditions (13) (14)).

Algorithm 5.1 (identification with the test signal amplitude tuning).

Step 1. Apply test signal (3) under $\rho = \rho^{(1)} = \frac{u^*}{2}$ to the plant (1), measure Fourier's filter outputs $\hat{\alpha}_k(\delta)$, $\hat{\beta}_k(\delta)$ $(k = \overline{1,n})$ for $\delta = \nu T^*$, where $\nu = 2, 3, \ldots, \nu^*$, $\nu^* = [\frac{\delta_b}{T^*}]$ (symbol $[\frac{a}{b}]$ is the least integer part of a ratio), solve the frequency equations (10) and check the conditions (13) (14) for each ν $(\nu = 2, \ldots, \nu^*)$.

Step 2. If the conditions (13) (14) are fulfilled then identification is ended, if the contrary is the case then go to step 1 placing $\rho = \rho^{(2)} = \frac{\rho^{(1)}}{2}$, ets.

If the conditions (30) are violated then the plant (1) is enclosed by the feedback (17) whose coefficients are found by means of the following algorithm.

Algorithm 5.2 (determination feedback coefficients): apply the test signal (3) under $\rho = \rho^{(1)} = \frac{u^*}{2}$ to the plant (1), measure Fourier's filter outputs $\hat{\alpha}_k(\delta_b)$ and $\hat{\beta}(\delta_b)$, solve the frequency equations (10) and identity (18), enclose the plant (1) by feedback and determine the unstability degree $\tilde{C}_0$. If $\tilde{C}_0 = 0$ then the process is ended, if the contrary is the case then go to the beginning and place $\rho = \rho^{(2)} = \rho^{(1)}/2$, ets.

If algorithm 5.2 is convergence to the estimates $\tilde{C}_0 = 0$ then the plant (1), enclosed by the feedback (17–19), is identified with a tuning of the amplitude $\tilde{\rho}$ on the basis of algorithm 5.1 which is used for "a plant" (21). The estimates $\hat{\alpha}_k$ and $\hat{\beta}_k$ $(k = \overline{1,n})$ in algorithm 5.1 are calculated by formulae (25) (26). If disturbance $f(t)$ does not contain the test frequencies then $\tilde{\lambda} = 0$.

6. EXAMPLE

Consider the completely controllable plant

$$\ddot{y} + d_1\dot{y} + d_0 y = k_1\dot{u} + k_0 u + f. \tag{31}$$

with unknown coefficients. It is known that the unstability degree estimate of the plant is $C_0 = 4.1$ and the boundaries of the disturbance and test signal are $f^* = 1$ $u^* = 25$.

Problem 6.1 Find the plant coefficients estimates such that the identification errors meet the demands

$$|\Delta d_i| \leq 1, \quad |\Delta k_i| \leq 1, \quad i = \overline{0,1}. \tag{32}$$

Remark 6.1 The true coefficients of the plant (31) $d_0 = -16$, $d_1 = 0$, $k_0 = -30$, $k_1 = 5$, the disturbance $f(t) = \sin 2.5t$, the initial conditions $y(t_0) = \dot{y}(t_0) = 0$, $t_0 = 0$. Minimum-phased version of this plant (it is a robot-cyclist) is described by (Fomin, *et al.* 1981).

The numerical experiments have been performed and they have been consisted of three groups: (a), (b) and (c).

(a). The plant (31) was excited by the test signal

$$u(t) = \rho e^{4.2t}(\sin 9t + \sin 18t) \tag{33}$$

To determine the amplitude ρ the algorithm 5.2 was used with polynomial $\psi(s) = 5s^3 + 100s^2 + 670s + 1500$, and the plant (31) was enclosed under $l_0 = 0$ by the feedback

$$g_1\dot{u} + g_0 u = r_1\dot{y} + r_0 y + l_0\tilde{u}. \tag{34}$$

As a result the amplitude $\rho = 0.03$ $(\delta_b = 2.09s)$ was found. The feedback coefficients (providing stability of the system (31) (34)) obtained under this amplitude are

$$g_0 = 728, \quad g_1 = 5, \quad r_0 = 461, \quad r_1 = 116. \tag{35}$$

(b). The system (31), (34), (35) under $l_0 = 100$ was excited by the test signal $\tilde{u}(t) = \tilde{\rho}(\sin 9 + \sin 18t)$, and in accordance with algorithms 5.1 the amplitude $\tilde{\rho} = 1$ $(\tilde{\delta} = 14.66$ s, $T^* = \frac{2\pi}{9}))$ was found. The plant coefficients obtained under this amplitude are

$$\hat{d}_0 = -16.2, \ \hat{d}_1 = -0.052, \quad \hat{k}_0 = -30.3, \ \hat{k}_1 = 5. \tag{36}$$

(c). For the model (31), (36) to validate the plant was enclosed by feedback (34) with $l_0 = 100$ and the coefficients

$$g_0 = 484, \quad g_1 = 5, \quad r_0 = 309, \quad r_1 = 76.7, \tag{37}$$

obtained from the identity (18) in which the polynomials $\hat{d}(s)$, $\hat{k}(s)$ had the coefficients (36) and $\psi(s)$ was took from experiment (a), Validation time $\tilde{\delta}$ was determined by conditions (29) under $\varepsilon_1^\theta = 0.0302$, $\varepsilon_2^\theta = 0.027$; $\varepsilon_1^\gamma = 0.125$, $\varepsilon_2^\gamma = 0.043$.

Under $\tilde{\delta} = 14.66$ s and $\tilde{\rho} = 1$ the following values were obtained $|\hat{\theta}_1 - \theta_1^*| = 0.0034$, $|\hat{\theta}_2 - \theta_2^*| = 0.0022$, $|\hat{\gamma}_1 - \gamma_1^*| = 0.00077$, $|\hat{\gamma}_2 - \gamma_2^*| = 0.0006$.

7. CONCLUSION

Algorithm 4.2 of a validation of the plant model obtained as a result of the finite-frequency identification is designed. It bases itself upon the assertion 4.1 about a connection between identification errors and deviations of feedback output FDP that are obtained experimentally from the numbers θ_k^* and γ_k^* that are calculated.

Two ways of identification for a bounded test signal are proposed: identification with tuning of signal amplitude (algorithm 5.1), and identification of a plant enclosed by the feedback (17)-(19) whose coefficients are determined on the basis of algorithm 5.2.

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APPENDIX

Proof of the assertion 4.1.

Form the transfer functions difference

$$w_u(s) - w_u^*(s) = \frac{l(s)}{\varphi(s)\psi(s)}(d(s)\psi(s) - \hat{d}(s)\varphi(s)) \quad (38)$$

Taking into account $\hat{d}(s) = d(s) - \triangle d(s)$, $\hat{k}(s) = k(s) - \triangle k(s)$ *one easily represents*

$$\begin{aligned} d(s)\psi(s) - \hat{d}(s)\varphi(s) &= d(s)\{[d(s)-\triangle d(s)]g(s)- \\ -[k(s)-\triangle k(s)]r(s)\} &- [d(s)-\triangle d(s)][d(s)g(s)- \\ -k(s)r(s)] &= r(s)[d(s)\triangle k(s) - k(s)\triangle d(s)] \end{aligned} \quad (39)$$

The equations (27) are obtained after substitution of expression (39) into the difference (38) under $s = \tilde{s}_k \ (k = \overline{1,n})$).

Proof of the assertion 5.1.

It may show that the function q_2 in the expression (8) is a linear function of the initial conditions $y^{(i)}(t_0)$ $(i = \overline{0, n-1}))$ and the value $f(t_0)$ and therefore

$$|q_2| \le \nu_1\varepsilon_0 + \nu_2 f^* \quad (40)$$

where ν_1 and ν_2 are some positive numbers.

Represent the searched values f^{**} and ε_0^* as linear functions of unknown amplitude ρ^*

$$f^{**} = \nu_3\rho^*, \quad \varepsilon_0^* = \nu_4\rho^* \quad (41)$$

Placing (40) in the sum (8) and replacing f^* and ε_0 by the expression (41) it easily see that the function $\nu(\delta)$ does not depend on $\rho = \rho^*$ and that is why for any ν_3 and ν_4 the time δ^* may be found. After placing the value δ^* into the inequality (16) a number $\rho^* \le \frac{u^*}{n} e^{-\lambda\delta^*}$ is determined.