REGECTION OF BOUNDED HARMONIC EXTERNAL DISTURBANCES WITH UNKNOUN FREQUENCIES AND AMPLITUDES

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Abstract: Problem of accurate control of steady-state of the linear time-invariant multivariable systems subject to bounded harmonic external disturbances of unknoun frequencies and amplitudes is formulated. The way of its solution on the base of H_∞ suboptimal control with appropriate choice of weighting matrices is proposed.

Keywords: Linear system, H_∞ suboptimal control, disturbance rejection.

1. INTRODUCTION

The work quality of a real control system is characterized by technical indices: a maximum overshoot, settling time, steady-state errors for each controlled variable, stability margins etc. These technical indices checked by the experiment are a language of task description on a design of any control system.

The accurate control of steady-state is a control that provides the specified tolerances on the steady-state errors of the controlled outputs in the presence of the unmeasured external disturbance and measurement noise with the known boundaries.

Note that the modern control theory uses axiomatic indices such a value of a quadratic functional, characteristic polynomial roots and the H_∞ norm of a transfer function matrix of closed-loop system that are axioms for LQ-optimization, modal control and H_∞ optimal control. These effective methods of synthesis may be used for real control system design if the connections between the technical and axiomatic indices are established. The first paper devoted by LQ-optimal system properties is the investigation [8] where it was shown that for any linear control system with state feedback (and, in particular, for a "bad" system in the sense of the technical indices) may be found the quadratic functional for which this system is optimal. An analogous results take place for a H_∞-optimal [9] and l_1-optimal control systems [4].

The aim of this paper is a investigation of the accuracy problem of steady-state of H_∞ suboptimal control. However, a standard H_∞ suboptimal control [5] deals with finite energy signals (external disturbance and measurement noise) only, hence, it has practical limitation when persistent disturbances are present [10, 13].

In many control problems there is a requirement to reject harmonic disturbances. Some examples where such harmonic disturbances may arise is in vibrations in rotating machinary with an unknown unmeasured angular velocity [12].

In this paper we consider accuracy properties of H_∞ suboptimal system under bounded harmonic disturbances of unknown frequencies and amplitudes. It is significant in order for the H_∞ control theory becomes a practicable tool for a control system design-engineer.

It should be noted that well known results (see, for example, [6, 12, 16, 17]) for rejection of harmonic disturbances are proposed when frequencies of the external disturbances are known. In this paper, however, we deals with sinusoids of unknown frequencies. In contrast to l_1-optimal control theory [10, 13] we consider steady-state errors only.

The main result of this paper is connection between steady-state errors and weighting matrices Riccati equations of H_∞ suboptimal control. The last is analogous to the results solving well known problem of coefficients choice of the optimization functional for LQ problem [1] and generalizes investigations [2].

2. PROBLEM STATEMENT

Consider the time-invariant system described by the following equations

$$\dot{x} = Ax + B_1 w + B_2 u, \quad z = C_1 x, \quad y = C_2 x + \eta, \quad (2.1)$$

$$\dot{x}_r = A_r x_r + B_r y, \quad u = C_r x_r + D_r y, \quad (2.2)$$

where $x(t) \in R^n$ is a state vector of the plant (2.1), $u(t) \in R^m$ is a control, $z(t) \in R^m$ is a controlled output, $y(t) \in R^{m_2}$ is a measured output, $w(t) \in R^\mu$ is an external unmeasured disturbance, $\eta(t) \in R^{m_2}$ is a measurement noise vector, $x_r(t) \in R^{n_r}$ is a state vector of the controller (2.2). The constant matrices A, B_1, B_2, C_1 and C_2 are known. The pairs (A, B_1) and (A, B_2) are stabilizable and the pairs (C_1, A) and (C_2, A) are detectable. A_r, B_r, C_r, D_r are unknown matrices.

The disturbance and noise components are the bounded functions that may be represented as

$$w_i(t) = \sum_{k=1}^{p} \left(\delta_i^{s,k} \sin \omega_k t + \delta_i^{c,k} \cos \omega_k t\right) \quad i = \overline{1, \mu}, \quad (2.3)$$

$$\eta_j(t) = \sum_{q=1}^{l} \left(\alpha_j^{s,q} \sin \tilde{\omega}_q t + \alpha_j^{c,q} \cos \tilde{\omega}_q t\right) \quad j = \overline{1, m_2}. \quad (2.4)$$

Here the amplitudes $\delta_i^{s,k}$, $\delta_i^{c,k}$ $\left(i = \overline{1, \mu}\right)$, $\left(k = \overline{1, p}\right)$; $\alpha_j^{s,q}$, $\alpha_j^{c,q}$ $\left(j = \overline{1, m_2}\right)$, $\left(q = \overline{1, l}\right)$ and the frequencies ω_k $\left(k = \overline{1, p}\right)$, $\tilde{\omega}_q$ $\left(q = \overline{1, l}\right)$ are unknown, however

$$\sum_{k=1}^{p} \left(|\delta_i^{s,k}|^2 + |\delta_i^{c,k}|^2\right) \le w_i^{*2} \quad i = \overline{1, \mu}, \quad (2.5)$$

$$\sum_{q=1}^{l} \left(|\alpha_j^{s,q}|^2 + |\alpha_j^{c,q}|^2\right) \le \eta_j^{*2} \quad j = \overline{1, m_2}, \quad (2.6)$$

where p and l are given integers; w_i^* $\left(i = \overline{1, \mu}\right)$, η_j^* $\left(j = \overline{1, m_2}\right)$ are the given numbers.

Determine steady-state errors of the system (2.1), (2.2) as

$$z_{i,st} = \lim_{t \to \infty} \sup |z_i(t)| \quad i = \overline{1, m_1} \quad (2.7)$$

and analogous determine steady-state values of the control as

$$u_{i,st} = \lim_{t \to \infty} \sup |u_i(t)| \quad i = \overline{1, m}. \quad (2.8)$$

Problem of accurate control of steady-state consists in finding of controller (2.2) such that the system (2.1), (2.2) satisfies the requirements on accuracy

$$z_{i,st} \le z_i^* \quad i = \overline{1, m_1}, \quad (2.9)$$

where z_i^* $\left(i = \overline{1, m_1}\right)$ are the specified numbers.

3. STEADY-STATE VALUES LEMMA

Let $T_{zw}(s)$ be a stable $(m_1 \times \mu)$ transfer function matrix from external signal w to output z ($\eta = 0$, for simplicity).

$$z(s) = T_{zw}(s) w(s), \quad (3.1)$$

and $\tilde{A}$, B, C are matrices its minimal state space realization

$$T_{zw}(s) = C(sI - \tilde{A})^{-1} B. \quad (3.2)$$

The next bounded real lemma (see, for example, [7, 11, 15]) is the direct corollary of the famous so-called Kalman-Yakubovich-Popov lemma: Let $Q = Q^T$ is any positive-definite matrix and $\gamma > 0$ is a given number, then the frequency inequality

$$T_{zw}^T(-j\omega) Q T_{zw}(j\omega) \le \gamma^2 I, \quad \omega \in [0, \infty) \quad (3.3)$$

holds iff exist positive-definite solution $P > 0$ of the quadratic matrix equation

$$\tilde{A}^T P + P\tilde{A} + \gamma^{-2} P B B^T P = -C^T Q C. \quad (3.4)$$

If $Q = I$ then aim condition of H_∞ suboptimal control theory follows [5] from inequality (3.3).

Now, consider physical sense of the frequency inequality (3.3) with respect introducing definition (2.7). Suppose that external signal w from (3.1) has a view (2.3), (2.5) and consider, for convenience, diagonal weighting matrix Q in (3.3): $Q = \mathrm{diag}[q_1, q_2, \ldots, q_{m_1}]$, $q_i > 0$ $\left(i = \overline{1, m_1}\right)$.

Introduce the vector $w^* = [w_1^*, w_2^*, \ldots, w_\mu^*]^T$ with components from the right side (2.5), and determine the standard Euclidean norm of this vector $\|w^*\| = \sqrt{(w^*)^T w^*}$. Then the following result takes place.

Lemma 1 (Steady-State Values Lemma). *Let the frequency inequality (3.3) holds. Then steady-state outputs of the system (3.1) satisfy the following inequality*

$$\sum_{i=1}^{m_1} q_i z_{i,st}^2 \leq \gamma^2 p \|w^*\|^2. \qquad (3.5) \blacksquare$$

Proof: Steady-state outputs of the system (3.1) are described as

$$\lim_{t\to\infty} z_i(t) = \sum_{k=1}^{p} a_i(\omega_k)\sin(\omega_k t + \phi_i(\omega_k)) \quad i = \overline{1, m_1}, \tag{3.6}$$

where $a_i(\omega_k) \geq 0$ and $\phi_i(\omega_k)$ $(i = \overline{1, m_1},\ k = \overline{1, p})$ are amplitudes and phases of the forced oscillations.

It is obviously that

$$z_{i,st} \leq \sum_{k=1}^{p} a_i(\omega_k) \quad i = \overline{1, m_1}. \tag{3.7}$$

Using the Coushy-Bunyakovsky inequality [3]

$$\left(\sum_{k=1}^{p} a_i(\omega_k)\right)^2 \leq p \sum_{k=1}^{p} a_i^2(\omega_k) \tag{3.8}$$

it is easily obtained on the base of (3.7) that

$$\sum_{i=1}^{m_1} q_i z_{i,st}^2 \leq p \sum_{k=1}^{p} \sum_{i=1}^{m_1} q_i a_i^2(\omega_k). \tag{3.9}$$

On the other hand, expression (3.1) and inequality (3.3) give

$$\sum_{i=1}^{m_1} q_i a_i^2(\omega_k) = \delta_-^{(k)^T} T_{zw}^T(-j\omega_k) Q T_{zw}(j\omega_k) \delta_+^{(k)} \leq$$

$$\leq \gamma^2 \delta_-^{(k)^T} \delta_+^{(k)} \quad k = \overline{1, p}, \tag{3.10}$$

where $\delta_+^{(k)} = [\delta_1^{(k)} e^{j\psi_{1k}},\ \delta_2^{(k)} e^{j\psi_{2k}},\ \ldots,\ \delta_\mu^{(k)} e^{j\psi_{\mu k}}]^T$,

$\delta_-^{(k)} = [\delta_1^{(k)} e^{-j\psi_{1k}},\ \delta_2^{(k)} e^{-j\psi_{2k}},\ \ldots,\ \delta_\mu^{(k)} e^{-j\psi_{\mu k}}]^T$,

$\delta_i^{(k)} = \sqrt{|\delta_i^{s,k}|^2 + |\delta_i^{c,k}|^2}$ $(i = \overline{1, \mu})$, $(k = \overline{1, p})$, $\psi_{ik} = \arcsin \delta_i^{c,k} / \delta_i^{(k)}$ $(i = \overline{1, \mu})$, $(k = \overline{1, p})$.

Lemma 1 follows from inequalities (3.9), (3.10) and (2.5). $\blacksquare$

Note that the estimation (3.5) is achievable.

4. STATE FEEDBACK

First consider case when the state vector of the plant (2.1) is available for feedback and the measurement noise is absent. It means that

$$y = x, \quad C_2 = I, \quad \eta = 0. \tag{4.1}$$

Introduce new controlled output

$$\begin{aligned} \overline{z} &= \overline{C}_1 x + D_{12} u, \\ \overline{C}_1^T &= [C_1^T Q^{1/2}, 0], \quad D_{12}^T = [0, R^{1/2}], \end{aligned} \tag{4.2}$$

where matrices $\overline{C}_1$ and D_{12} satisfy the following property

$$D_{12}^T[\overline{C}_1,\ D_{12}] = [0, R], \tag{4.3}$$

and $Q = Q^T$, $R = R^T$ are any positive-definite matrices.

Consider the next problem: find the state feedback control law

$$u = D_r x, \tag{4.4}$$

such that the following condition holds

$$T_{\overline{z}w}^T(-j\omega) T_{\overline{z}w}(j\omega) \leq \gamma^2 I, \quad \omega \in [0, \infty) \tag{4.5}$$

where $T_{\overline{z}w}(s) = (\overline{C}_1 + D_{12} D_r)(sI - A - B_2 D_r)^{-1} B_1$ and $\gamma > 0$ is a given number.

It is well known standard H_∞ suboptimal control problem [5]. The solution this problem on the base the bounded real lemma was derived, for example, in [14].

The control law has a view

$$u = D_r x, \quad D_r = -R^{-1} B_2^T P, \tag{4.6}$$

where $P = P^T \geq 0$ satisfied the algebraic Riccati equation (ARE)

$$A^T P + PA - PB_2 R^{-1} B_2^T P + \gamma^{-2} P B_1 B_1^T P = -C_1^T Q C_1, \tag{4.7}$$

which differs from usual ARE H_∞ suboptimal control theory [5] by weighting matrices $Q \neq I$ and $R \neq I$.

These weighting matrices play important role for main results of this paper. It will be shown in the sequel that accuracy of closed-loop system (2.1) ,(4.6) is depend on the choice of matrices Q and R.

In linear-quadratic optimal control theory so-called optimal frequency condition (circle condition [8]) for transfer matrix of the open-loop system plays an important role [1, 8].

An analogous condition may be derived in H_∞ suboptimal control theory, which is called γ-optimal condition in the frequency domain. The main difference of this condition from circle condition consists in that the former includes transfer matrices of the closed-loop system (2.1), (4.6).

To obtain γ-optimal condition denote $\bar{A} = A + B_2 D_r$ and determine the following closed-loop transfer matrices

$$T_{zw}(s) = C_1(sI - \bar{A})^{-1}B_1, \tag{4.8}$$

$$T_{uw}(s) = D_r(sI - \bar{A})^{-1}B_1, \tag{4.9}$$

$$T_w(s) = \gamma^{-2}{B_1}^T P(sI - \bar{A})^{-1}B_1, \tag{4.10}$$

which connect external disturbance w with initial controlled output z, control u and vector of the "worst" disturbance in the $L_2[0,\infty)$ sense [5].

Theorem 1. *The transfer matrices (4.8)-(4.10) of closed-loop suboptimal system (2.1), (4.6) satisfy the following identity (γ- optimal condition in the frequency domain)*

$$[I - T_w(-j\omega)]^T\gamma^2[I - T_w(j\omega)] = \gamma^2 I - T_{zw}^T(-j\omega)QT_{zw}(j\omega) -$$

$$-T_{uw}^T(-j\omega)RT_{uw}(j\omega), \quad \omega \in [0,\infty). \tag{4.11}$$ ■

Proof: Using a closed-loop system matrix $\bar{A} = A + B_2 D_r$ the equation (4.7) is represented as

$$\bar{A}^T P + P\bar{A} + \gamma^{-2}PB_1B_1^TP + PB_2R^{-1}B_2^TP = -C_1^TQC_1.$$

From the last equation after adding and subtraction sP and multiplication of the obtained expression by $B_1^T(-sI - \bar{A}^T)^{-1}$ and $(sI - \bar{A})^{-1}B_1$ from the left and from the right respectively the required result follows if connections (4.6),(4.8)-(4.10) take into account. ■

Corollary 1. *The transfer matrices (4.8), (4.9) of the closed-loop H_∞ suboptimal system (2.1), (4.6) satisfy the following frequency inequality*

$$T_{zw}^T(-j\omega)QT_{zw}(j\omega) + T_{uw}^T(-j\omega)RT_{uw}(jw) \le \gamma^2 I,$$

$$\omega \in [0,\infty) \tag{4.12}$$ ■

Proof: It is obvious since the left part of identity (4.11) is a positive semi-definite matrix, hence, the right part gives the required result. ■

Note that the inequality (4.12) is merely another form of the aim condition (4.5).

Let the weighting matrices Q and R from ARE (4.7) be

$$Q = \mathrm{diag}[q_1,\ q_2,\ \ldots,\ q_{m_1}], \quad q_i > 0 \quad i = \overline{1, m_1}, \tag{4.13}$$

$$R = \mathrm{diag}[r_1,\ r_2,\ \ldots,\ r_m], \quad r_i > 0 \quad i = \overline{1, m}. \tag{4.14}$$

Then the following theorem takes place.

Theorem 2. *The steady-state errors and steady-state values of the control components of the closed-loop H_∞ suboptimal system (2.1), (4.6) satisfy the inequality*

$$\sum_{i=1}^{m_1} q_i z_{i,st}^2 + \sum_{i=1}^{m} r_i u_{i,st}^2 \le \gamma^2 p\|w^*\|^2. \tag{4.15}$$ ■

Proof: This result is the direct corollary of the lemma 1 and follows from frequency inequality (4.12). ■

Note that the estimation (4.15) is achievable and may be used in order to choice weighting coefficients of the matrices Q and R such that controller (4.6) provides a accuracy requirement (2.9). In particular, let weighting coefficients be

$$q_i \ge \frac{p\|w^*\|^2}{(z_i^*)^2} \quad i = \overline{1, m_1}; \quad r_i > 0 \quad i = \overline{1, m}, \tag{4.16}$$

and denote γ^* a minimal value γ for that a solution $P \ge 0$ of ARE (4.7) exists.

Corollary 2. *Let the weighting matrices (4.13), (4.14) satisfy the inequalities (4.16). Then steady-state errors of the closed-loop H_∞ suboptimal system (2.1), (4.6) satisfy the following inequality*

$$\sum_{i=1}^{m_1}\left(\frac{z_{i,st}}{z_i^*}\right)^2 < \gamma^{*2}. \tag{4.17}$$ ■

Proof: This result is the direct corollary of the inequalities (4.15) and (4.16). ■

So, from (4.17) we now conclude that the steady-state errors of the closed-loop system (2.1), (4.6) satisfy the following inequalities

$$z_{i,st} < \gamma^* z_i^* \ \left(i = \overline{1, m_1}\right). \tag{4.18}$$

Note, that if $B_1 = B_2$ then an ARE (4.7) for all $\gamma^2 > r_i = 1 \ \left(i = \overline{1, m}\right)$ is the usual ARE of the LQ optimal control theory and its solution $P \ge 0$ exist. In this case $\gamma^* \to 1$, hence, inequalities (2.9) are fulfilled.

5. OUTPUT FEEDBACK

Let the measured output of the plant (2.1) be

$$y = C_2 x + \eta. \tag{5.1}$$

Introduce the vector $\bar{w} = [w^T, \ \eta^T]^T$ and find the controller (2.2) such that the following aim condition holds

$$T^T_{\bar{z}\bar{w}}(-j\omega)T_{\bar{z}\bar{w}}(j\omega) \leq \gamma^2 I, \quad \omega \in [0, \infty), \qquad (5.2)$$

where $T_{\bar{z}\bar{w}}(s)$ is the closed-loop transfer matrix of the system (2.1), (2.2) from signal $\bar{w}$ to controlled output $\bar{z}$ (4.2), γ is a given number.

It is well known standard H_∞ suboptimal output feedback problem [5]. The solution of this problem on the base the bounded real lemma was derived in paper [14].

The control law is described by

$$u = -R^{-1}B_2^T P x_r, \qquad (5.3)$$

where $P = P^T \geq 0$ is the positive semi-definite solution ARE (4.7), x_r is the state vector of the observer

$$\dot{x}_r = Ax_r + B_2 u + B_1 w_r + K_f(y - C_2 x_r), \qquad (5.4)$$

in which

$$w_r = \gamma^{-2} B_1^T P x_r, \qquad (5.5)$$

is the estimation of the disturbance and K_f is the observer gain matrix

$$K_f = (I - \gamma^{-2} YP)^{-1} Y C_2^T, \qquad (5.6)$$

where $Y \geq 0$ is a positive semi-definite solution ARE:

$$AY + YA^T + \gamma^{-2} Y C_1^T Q C_1 Y - Y C_2^T C_2 Y = -B_1 B_1^T, \quad (5.7)$$

and he following condition must be fulfilled

$$\lambda_{\max}(YP) < \gamma^2, \qquad (5.8)$$

where $\lambda_{\max}(M)$ is a maximum eigenvalue of a matrix M.

Write the transfer matrix $T_{\bar{z}\bar{w}}(s)$ in view (4.2) as

$$T_{\bar{z}\bar{w}}(s) = \begin{bmatrix} Q^{\frac{1}{2}} T_{z\bar{w}}(s) \\ R^{\frac{1}{2}} T_{u\bar{w}}(s) \end{bmatrix} \qquad (5.9)$$

where $T_{z\bar{w}}(s)$ and $T_{u\bar{w}}(s)$ are matrices of the closed-loop system (2.1), (5.3)-(5.7) from $\bar{w}$ to z and from $\bar{w}$ to u accordingly.

Then frequency inequality (5.2) may be represented as

$$T^T_{z\bar{w}}(-j\omega) Q T_{z\bar{w}}(j\omega) + T^T_{u\bar{w}}(-j\omega) R T_{u\bar{w}}(j\omega) \leq \gamma^2 I$$
$$\omega \in [0, \infty) \qquad (5.10)$$

Let weighting matrices Q and R have a diagonal form (4.13), (4.14).

Theorem 3. *The steady-state errors and steady-state values of the control components of the closed-loop H_∞ suboptimal system (2.1), (5.3)-(5.7) satisfy the following inequality*

$$\sum_{i=1}^{m_1} q_i z^2_{i,st} + \sum_{i=1}^{m} r_i u^2_{i,st} \leq \gamma^2 (p + l) \|\bar{w}^*\|^2, \qquad (5.11)$$

where $\|\bar{w}^*\|$ is a Euclidean norm of the vector $\bar{w}^* = [w_1^*, \ w_2^*, \ \ldots, \ w_\mu^*, \ \eta_1^*, \ \ldots, \ \eta_{m_2}^*]^T$ whose components are took from the right parts inequalities (2.5), (2.6). ■

Proof: The result follows from frequency inequality (5.10) after application of lemma 1. ■

Note, that if diagonal elements of the weighting matrices Q and R satisfy the following conditions

$$\begin{matrix} q_i \geq \dfrac{(p + l)\|\bar{w}^*\|^2}{(z_i^*)^2} & i = \overline{1, m_1}; \\ r_i > 0 \quad i = \overline{1, m}, & \end{matrix} \qquad (5.12)$$

then next result solving the problem of accurate control is true.

Corollary 3. *Let the weighting matrices (4.13), (4.14) satisfy the inequalities (5.12). Then steady-state errors of the closed-loop H_∞ suboptimal system (2.1), (5.3)-(5.7) satisfy inequality (4.17).*

Proof: This result follows from inequalities (5.11) and (5.12). ■

So, the steady-state errors of the closed-loop system (2.1), (5.3)-(5.7), (5.12) satisfy inequality (4.18) , where γ^* is a minimal value γ for that the solutions $P \geq 0$ and $Y \geq 0$ of AREs (4.7), (5.7) are exists and condition (5.8) holds.

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