

ADAPLAB-M: IDENTIFICATION AND ADPTATION TOOLBOX FOR MATLAB

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Abstract: Description of ADAPLAB-M (MATLAB ToolBox) for identification and adaptation is given. As distinct from known MATLAB ToolBoxes, ADAPLAB-M algorithms proceeds from assumption that an external disturbance applied to a plant and a measurement noise are unknown-but-bounded functions. A test signal as a sum of a minimal number of harmonics is used for identification. An adaptive control is formed on the base of H_∞ -optimization and the results of identification of the plant and a closed-loop system.

Keywords: Identification, adaptive control, MIMO, linear systems, frequency domain, unknown-but-bounded disturbance, software tool.

1. INTRODUCTION

The software of identification is a active developing direction of the software for automatic control systems designing. It is called both practical significance of identification as and fast development of its theory.

The theory of identification is developing in several directions. One of these is dedicated to a attenuation of limitations on a view of an external disturbances and measurement noise, which are realized in an identification process. In this direction a finite-frequency method of identification and adaptive control (Alexandrov and Orlov, 2002) and method of recurrent target inequalities for an adaptive control (Yakubovich, 1988) are developed. In these methods the external disturbance and measurement noise are assumed almost arbitrary.

In the software of identification MATLAB Toolboxes are well known. MATLAB Toolboxes con-

tain the software of methods of least squares and frequency domain methods and also method of instrument variables. The external disturbance and the measurement noise are guessed the white noise processes or it is supposed that the disturbances and the controlled inputs are not correlated.

In practice, these suppositions often are violated and for such cases serves package ADAPLAB (Alexandrov and Orlov, 1995), which is based on a method of finite-frequency identification.

ADAPLAB-M, being a development of a package ADAPLAB, is designed in medium MATLAB.

To describe features of construction of ADAPLAB-M we shall introduce a concept the gamma and matlab-structures. It is two structure of the computer aided design. They are oriented on two user groups of the software. The first group are the researchers, who well know the control theory and problem of control of a concrete data domain (aircraft, power

engineering, robotics etc.). The matlab-structure is oriented on this user group. It contains a rich spectrum of functions (m-files), using which the researcher easily creates the program for the solution of real problems of its data domain.

The second user group are the engineers-developers of a control system (more precisely, engineers-developers of control algorithms of the control system). The purposes of this group and a small time for Control System development eliminate a capability of their participation in creation of the software for the solution of their problem. Besides, the necessity of a profound knowledge of the theory of control also handicaps their work with the matlab-structure.

The gamma-structure is intended for this user group. Its basis is a directive. The directive is a program consisting of three parts: a software for user interface, computational part and means for a of output of intermediate and final results. Each directive decides the definite class of problems of a designing of control algorithm.

The developer of control system, working into gamma-structure, selects from a list the directive, which decides his problem, and enters by a "natural" language the description of his problem. The solution of the problem is implemented automatically. Analyzing the results he makes a decision on an acceptability of outcome.

The gamma-structure has been of a base of Gamma-systems (Gamma-1, Gamma-1M, Gamma-1PC (Alexandrov and Panin, 1997) etc.) is developing already more than 30 years.

ADAPLAB-M is constructed in medium MATLAB pursuant to the gamma-structure. It consists of two directives "finite-frequency identification" and " H_∞ -optimal adaptive control", which are MATLAB-functions. The computational parts of these directives consist of a number of MATLAB-functions of Control System Toolbox, and also recreated and hooked up MATLAB-functions, such as formation and solution of frequency equations, Fourier filter etc.

The directives serve both for planning of experiment and for implementation of identification and adaptation in the real time. The stage of planning of experiment serves for definition of the tuned parameters of algorithms of identification and adaptation such as duration of identification, amplitudes and frequencies of a test signal etc. Definition of these parameters allow to reduce duration of processes of identification and adaptation for a real plant of control (controlled process).

At a stage of planning of experiment the techno-

logical model of the plant is used. This model describes (in the form of factors of differential equations of the plant, parameters of disturbances) a knowledge of the specialist of the plant (technologist of process). It is some suspected model of the true plant. It can essentially differ from true model of the plant. However, the parameters of algorithms of identification and adaptation obtained by its help are the first their values in real experiments.

2. DIRECTIVE M111: FINITE-FREQUENCY IDENTIFICATION

2.1 Area of application.

Consider a linear time-invariant plant described by the following equations

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} + M\mathbf{f}, \quad \mathbf{y} = C\mathbf{x} + D\mathbf{u} + N\boldsymbol{\eta}, \quad t \geq t_0, \quad (1)$$

where $\mathbf{x}(t) \in \mathbf{R}^n$ is a state vector, $\mathbf{y}(t) \in \mathbf{R}^r$ is a measured output, $\mathbf{u}(t) \in \mathbf{R}^m$ is an input to be controlled, $\mathbf{f}(t) \in \mathbf{R}^\mu$ is an external disturbance and $\boldsymbol{\eta}(t) \in \mathbf{R}^s$ is a measurement noise. Components of the last two vectors are unknown-but-bounded functions: $|f_i(t)| \leq f_i^*$ and $|\eta_i(t)| \leq \eta_i^*$, where f_i^* ($i = \overline{1, \mu}$) η_i^* ($i = \overline{1, s}$) are positive numbers A , B , C and D are unknown constant matrices. The pair (A, B) is controllable and pair (A, C) is observable.

Transfer function of plant (1) is $W(s) = C(Es - A)^{-1}B + D$ and its elements are represented as

$$w_{ij}(s) = K_{ij}^{(0)} s^? \cdot \frac{\prod_{k=1}^{\bar{p}_{ij}} (\tilde{T}_{ij}^{(k)} s + 1) \prod_{k=1}^{\bar{p}_{ij}} (\tilde{T}_{ij}^{(k)2} s^2 + 2\tilde{T}_{ij}^{(k)} \tilde{\xi}_{ij}^{(k)} s + 1)}{\prod_{k=1}^{\bar{p}_{ij}} (\tilde{T}_{ij}^{(k)} s + 1) \prod_{\substack{k=1 \\ i = \overline{1, r} \quad j = \overline{1, m}}}^{\bar{p}_{ij}} (\tilde{T}_{ij}^{(k)2} s^2 + 2\tilde{T}_{ij}^{(k)} \tilde{\xi}_{ij}^{(k)} s + 1)} \quad (2)$$

Problem is to find coefficients estimates of these transfer functions.

2.2 Matlab-functions of the directive.

To identify plant (1) it is excited by the following test signals

$$\mathbf{u}_j(t) = \sum_{k=1}^{\varrho} \rho_{jk} \sin \omega_k t \cdot \mathbf{e}_j, \quad (3)$$

$$t_0 + (j-1)\tau \leq t < t_0 + j\tau, \quad j = \overline{1, m},$$

where ρ_{jk} ($j = \overline{1, m}$, $k = \overline{1, \varrho}$) is a specified amplitude of the k -th harmonic for the j -th experiment, ω_k ($k = \overline{1, \varrho}$) is a specified test frequency

$[\omega_k \neq 0 \ (k = \overline{1, \varrho}) \text{ and } \omega_i \neq \omega_j \ (i \neq j)],$
 $e_j = \text{col}_j E_m$ is the j -th of identity matrix E_m ,
 $\varrho = \nu + 1$, ν is an observability index of the plant
(it is determined below), τ is a specified duration
of the j -th experiment.

Test signals (3) are formed by two functions
TimeNet (“create a net of the time”) and Test
(“create a test signal”):

$\mathbf{t} = \text{TimeNet}(\text{number}, \text{Tdelay}, \text{Tfilter}, \text{Ndiv}, \boldsymbol{\omega})$

where **number** is a number of a time interval,
Tdelay and **Tfilter** are a filtration start
time and a filtration time (they are given as num-
bers of periods of the minimal test frequency),
Ndiv is a number of division of the maximal
test frequency period (it means that the sample
interval is $h = \frac{2\pi}{\max\{\omega_k\} \cdot \text{Ndiv}}$, **Ndiv** is a suf-
ficiently large number), **omega** is a vector of the
frequencies $\omega_k \ (k = \overline{1, \varrho})$ of the test signals (3),

$\text{gen} = \text{Test}(\text{rho}, \text{lambda}, \boldsymbol{\omega}, \mathbf{t}),$

where **rho** is a vector of test signal amplitudes
 $\rho_k \ (k = \overline{1, \varrho})$, $\lambda = 0$.

The external disturbance and measurement noise
are formed by function Dist (“create a distur-
bance”):

$\mathbf{f} = \text{Dist}(\text{par}, \mathbf{t}),$

where **par** is their parameters.

Solution of the equation (1) is carried out by func-
tion Lsim of Control System Toolbox.

Plant outputs $\mathbf{y}_j(t) \ (j = \overline{1, m})$ are applied to in-
puts of the Fourier’s filter

$$\begin{aligned} \hat{\alpha}_{ijk} &= \frac{2}{\rho_{jk}\tau} \int_{t_0+(j-1)\tau}^{t_0+j\tau} y_{ji}(t) \sin \omega_k(t-t_0) dt \\ \hat{\beta}_{ijk} &= \frac{2}{\rho_{jk}\tau} \int_{t_0+(j-1)\tau}^{t_0+j\tau} y_{ji}(t) \cos \omega_k(t-t_0) dt \end{aligned} \quad (4)$$

whose outputs give estimates of elements: α_{ijk}
 β_{ijk} of matrices $\hat{\mathbf{A}}_k = \text{Re} W(j\omega_k)$ and $\hat{\mathbf{B}}_k =$
 $\text{Im} W(j\omega_k) \ (k = \overline{1, \varrho})$, which are named fre-
quency domain parameters (FDP) of plant (1).

Corresponding function Fourier “calculate fre-
quency domain parameters” is

$[\text{alf}, \text{bet}] = \text{Fourier}(\text{rho}, \text{lambda}, \boldsymbol{\omega},$
 $\text{Tdelay}, \text{Tfilter}, \text{Ndiv}, \mathbf{y}, \mathbf{t})$

Using these estimates of frequency domain pa-
rameters, frequency equations of identification

(Alexandrov and Orlov, 2002) are solved by func-
tion FrId (“solution of frequency equations”):

$[\mathbf{P}, \mathbf{Q}] = \text{FrId}(\text{key}, \text{nu}, \mathbf{s}, \mathbf{W})$

where **nu** is a vector of observability indeces ν_i
($i = \overline{1, r}$), **s** is a vector wich complex compo-
nents: $s_k = \lambda + j\omega_k \ (k = \overline{1, \varrho})$, **W** is a com-
plex $\varrho r \times m$ matrix $[\widehat{\mathcal{W}}_1; \widehat{\mathcal{W}}_2; \dots; \widehat{\mathcal{W}}_\varrho]$, where
 $\widehat{\mathcal{W}}_k = \hat{\mathbf{A}}_k + j\hat{\mathbf{B}}_k \ (k = \overline{1, \varrho})$, **P** and **Q** are poly-
nomial matrices of the plant model in a form of
“input-output”: $P(s)\mathbf{y} = Q(s)\mathbf{u}$.

Function NuFDP (“Calculation of observability
indeces”) serves for determination of a vector

$\text{nu} = \text{NuFDP}(\mathbf{n}, \mathbf{s}, \mathbf{W})$

Function Cauchy (“convert from the “input-
output” model to a state-space model in the Lu-
enberger’s canonical form”) is

$[\mathbf{A}_K, \mathbf{B}_K, \mathbf{C}_K, \mathbf{D}_K] = \text{Cauchy}(\text{nu}, \mathbf{P}, \mathbf{Q})$

Using these matrices, the searched estimates of
transfer matrix of the plant $\widehat{W}(s)$ by a function
of model conversion from **ss**-model to **tf**-model
(Control System Toolbox) is found: $\widehat{W}(s) =$
 $\text{tf}(\text{ss}(\mathbf{A}_K, \mathbf{B}_K, \mathbf{C}_K, \mathbf{D}_K))$.

A number of functions are developed for the stage
of experiment planning: Canon (“convert from a
state-space model to a state-space model in the
Luenberger’s canonical form”), FDP (“calculate
the FDP matrices on the base of a state-space
model”), NuCauchy (“calculate the observability
indices of a state-space model”) and so on.

In addition, to analyze the identification results
are used the functions of Control System Toolbox:
Bode and zpk.

So, results of the directive **M111** are: transfer
matrix $\widehat{W}(s)$ of plant (1) and matrices $\hat{\mathbf{A}}_K$, $\hat{\mathbf{B}}_K$,
 $\hat{\mathbf{C}}_K$ and $\hat{\mathbf{D}}_K$ of its state-space model in Lu-
enberger’s canonical form.

Interface of the directive is formed by function
..... ()

Interface is shown in Fig. 1. For simplicity, the
technological model is entered by matrices A , B ,
 C and D . In future, the interface of the gamma-
structure, in which such models are entered by
“natural” differential equaions, will be developed.

An output of the intermediate and final result is
implemented by the functions of “input-output”
information.

3. DIRECTIVE M317:

3.1 Area of application.

Consider a linear time-invariant system described by the following equations

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}(\mathbf{u} + \Psi \mathbf{f}), \quad \mathbf{z} = \Phi \mathbf{C}\mathbf{x}, \quad \mathbf{y} = \mathbf{C}\mathbf{x} + \boldsymbol{\eta}, \quad t \geq t_0 \quad (5) \\ \dot{\mathbf{x}}_c &= \mathbf{A}_c \mathbf{x}_c + \mathbf{B}_c \mathbf{y}, \quad \mathbf{u} = \mathbf{C}_c \mathbf{x}_c, \quad t \geq t_N, \quad (6) \end{aligned}$$

where $\mathbf{x}_c(t) \in \mathbf{R}^n$ is a state vector of controller (6), $\mathbf{z}(t) \in \mathbf{R}^l$ is a controlled output, constant matrices of the system are unknown, except some specified matrices Ψ Φ .

It needs to find matrices \mathbf{A}_c , \mathbf{B}_c , \mathbf{C}_c of controller (6) such that, with a moment t_N , H_∞ -norm of system (5), (6) is minimal.

The controller (6) is a result of adaptation of a controller described by the following equations with piecewise-constant coefficients

$$\begin{aligned} \dot{\mathbf{x}}_c &= \mathbf{A}_c^{[\kappa]} \mathbf{x}_c + \mathbf{B}_c^{[\kappa]} \mathbf{y} + \mathbf{L} \mathbf{v}^{[\kappa]}, \quad \mathbf{u} = \mathbf{C}_c^{[\kappa]} \mathbf{x}_c, \quad (7) \\ t_{\kappa-1} &\leq t < t_\kappa \quad \kappa = \overline{1, N}, \end{aligned}$$

where κ is an adaptation interval number ($\kappa = \overline{1, N}$), t_κ is κ -th, t_κ as well as a number N and matrices $\mathbf{A}_c^{[\kappa]}$, $\mathbf{B}_c^{[\kappa]}$ $\mathbf{C}_c^{[\kappa]}$ are found during adaptation process, \mathbf{L} is a given matrix, $\mathbf{v}^{[\kappa]}(t) \in \mathbf{R}^m$ is the test signal.

Problem is to find an adaptation algorithm for coefficients of controller (7) to the coefficients of the H_∞ -optimal controller (6).

3.2 Matlab-functions of the directive.

On the first interval of adaptation the plant (5) is identified by directive **M111**, which gives matrices $\mathbf{A}^{[1]} = \hat{\mathbf{A}}_K$, $\mathbf{B}^{[1]} = \hat{\mathbf{B}}_K$ and $\mathbf{C}^{[1]} = \hat{\mathbf{C}}_K$.

On the base of these matrix the Riccati equations are formed and, making use of procedure of the H_∞ -optimal control design, matrices of a controller

$$\dot{\mathbf{x}}_c = \mathbf{A}_c^{[2]} \mathbf{x}_c + \mathbf{B}_c^{[2]} \mathbf{y} + \mathbf{L} \mathbf{v}^{[2]}, \quad \mathbf{u} = \mathbf{C}_c^{[2]} \mathbf{x}_c, \quad (8)$$

for the second interval of adaptation are calculated.

For this objective a function **ContRic** (" H_∞ -suboptimal control design") serves:

$$[\mathbf{Ac}, \mathbf{Bc}, \mathbf{Cc}, \mathbf{Dc}, \mathbf{gamma}] = \text{ContRic}(\mathbf{A}, \mathbf{B1}, \mathbf{B2}, \mathbf{C1}, \mathbf{C2}, \mathbf{alpha}, \mathbf{beta}, \mathbf{Q0}, \mathbf{Q1}, \mathbf{R1}, \mathbf{R2}),$$

where $\mathbf{A} = \mathbf{A}^{[1]}$ $\mathbf{B1} = \mathbf{B}^{[1]} \Psi$, $\mathbf{B2} = \mathbf{B}^{[1]}$, $\mathbf{C1} = \Phi \mathbf{C}^{[1]}$, $\mathbf{C2} = \mathbf{C}^{[1]}$, **alpha**, **beta** are scale

factors, **Q0**, **Q1**, **R1**, **R2** are weighting matrices. The scale vectors and weighting matrices are prescribed by an user. In particular, they may be calculated on the base of a required precision of control (Alexandrov and Chestnov, 1997).

The equations of system (5), (8), after eliminating variable $\mathbf{u}(t)$, are rewritten as

$$\begin{aligned} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{x}}_c \end{bmatrix} &= \begin{pmatrix} \mathbf{A} & \mathbf{B}\mathbf{C}_c^{[2]} \\ \mathbf{B}_c^{[2]}\mathbf{C} & \mathbf{A}_c^{[2]} \end{pmatrix} \times \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_c \end{bmatrix} + \\ &+ \begin{pmatrix} \mathbf{B}\Psi & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_c^{[2]} \end{pmatrix} \times \begin{bmatrix} \mathbf{f} \\ \boldsymbol{\eta} \end{bmatrix} + \begin{pmatrix} \mathbf{0} \\ \mathbf{L} \end{pmatrix} \cdot \mathbf{v}^{[2]}, \\ \mathbf{y} &= (\mathbf{C} \quad \mathbf{0}) \times \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_c \end{bmatrix} + (\mathbf{0} \quad \mathbf{E}_r) \times \\ &\times \begin{bmatrix} \mathbf{f} \\ \boldsymbol{\eta} \end{bmatrix}, \quad \mathbf{z} = (\Phi \mathbf{C} \quad \mathbf{0}) \times \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_c \end{bmatrix}. \quad (9) \end{aligned}$$

This new "plant" is excited by the following test signal

$$\begin{aligned} \mathbf{v}_j^{[2]}(t) &= \sum_{k=1}^{\bar{\varrho}} \bar{\rho}_{jk} \sin \bar{\omega}_k t \cdot \mathbf{e}_j, \\ t_1 + (j-1)\tau^{[2]} &\leq t < t_1 + j\tau^{[2]} \quad j = \overline{1, m}, \quad (10) \end{aligned}$$

where $\bar{\rho}_{jk}$ ($j = \overline{1, m}$) and $\bar{\omega}_k$ ($k = \overline{1, \bar{\varrho}}$) are amplitudes and frequencies of the test signals [$\bar{\omega}_k \neq 0$ ($k = \overline{1, \bar{\varrho}}$) $\bar{\omega}_i \neq \bar{\omega}_j$ ($i \neq j$)], $\bar{\varrho} = \bar{\nu} + 1$, $\bar{\nu} = \max\{\bar{\nu}_1, \bar{\nu}_2, \dots, \bar{\nu}_r\}$ is an observability index of system (9). The test signal is formed by function **Test**.

Identifying the "plant" by the directive **M111** the coefficient estimates of its transfer matrix $W_{yv}^{[2]}(s)$ are found.

In order to determine a finish moment of adaptation a hypothetical model of the closed-loop system is formed. This model has the view (9), where $\mathbf{A} = \mathbf{A}^{[1]}$, $\mathbf{B} = \mathbf{B}^{[1]}$ $\mathbf{C} = \mathbf{C}^{[1]}$. Its transfer matrix is $\bar{W}_{yv}(s)$.

Adaptation process is ended if the coefficients of the transfer matrices $W_{yv}^{[2]}(s)$ and $\bar{W}_{yv}(s)$ are close.

In the contrary is the case the matrices $\hat{\mathcal{V}}_k$ and $\hat{\mathcal{M}}_k$ of the closed-loop FDP estimates ($\hat{\mathcal{V}}_k = \text{Re } W_{yv}^{[2]}(j\bar{\omega}_k)$ and $\hat{\mathcal{M}}_k = \text{Im } W_{yv}^{[2]}(j\bar{\omega}_k)$) are used to find more precise matrices $\hat{\mathcal{A}}_k$ $\hat{\mathcal{B}}_k$ ($k = \overline{1, \bar{\varrho}}$) of the plant FDP estimates. For this objective it serves the following almost obvious link

$$\begin{aligned} \hat{\mathcal{A}}_k + j\hat{\mathcal{B}}_k &= [\mathcal{V}_k + j\mathcal{M}_k] \{W_c(j\bar{\omega}_k) \cdot \\ &\cdot [\mathcal{V}_k + j\mathcal{M}_k] + W_v(j\bar{\omega}_k)\}^{-1} \quad k = \overline{1, \bar{\varrho}}, \quad (11) \end{aligned}$$

where $W_c(s) = \mathbf{C}_c^{[2]} (\mathbf{E}_n s - \mathbf{A}_c^{[2]})^{-1} \mathbf{B}_c^{[2]}$ and

$$W_v(s) = C_c^{[2]} \left(E_n s - A_c^{[2]} \right)^{-1} L.$$

Calculation by formula (11) carries out by function Recalc ("convert from the closed-loop system FDP to the plant FDP"):

$$W = \text{Recalc}(s, Wcl, Ac, Bc, Cc, Dc, L),$$

where Wcl is a set of the closed-loop FDP matrices estimates.

Using the new matrices \hat{A}_k and \hat{B}_k ($k = \overline{1, \varrho}$) new matrices $A^{[2]}$, $B^{[2]}$, $C^{[2]}$ of the plant are found by function Frid, than the controller matrices $A_c^{[3]}$, $B_c^{[3]}$, $C_c^{[3]}$ are calculated by function ContRic and so on.

After ending of adaptation process (in moment t_N), the controller is described by the equation (6), where $A_c = A_c^{[N]}$, $B_c = B_c^{[N]}$ and $C_c = C_c^{[N]}$.

Directive **M317** serves too for a validation of a model derived by the directive **M111**, since nearness of the matrices $W_{yv}^{[2]}(s)$ and $\bar{W}_{yv}(s)$ means an identification accuracy (Alexandrov, 1996).

4. APPLICATION OF DIRECTIVE M111: GYROPLATFORM IDENTIFICATION

Consider gyro stabilized platform (Alexandrov and Chestnov, 1998) described by the following equations

$$\begin{aligned} p_1 \ddot{\beta}_1 - p_1 s_1 \dot{\alpha}_1 + p_1 c_1 \dot{\alpha}_2 - c_1 h_1 \alpha_1 - s_1 h_1 \alpha_2 + n_1 \dot{\beta}_1 &= 0, \\ p_2 \ddot{\beta}_2 + p_2 c_2 \dot{\alpha}_1 + p_2 s_2 \dot{\alpha}_2 - s_2 h_2 \alpha_1 + c_2 h_2 \alpha_2 + n_2 \dot{\beta}_2 &= 0, \\ j_x \dot{\alpha}_1 - (s_1 c_1 h_1 - s_2 c_2 h_2) \alpha_1 + (s_2^2 h_2 + c_1^2 h_1) \alpha_2 + \\ + (s_1 n_1 + c_1 h_1) \dot{\beta}_1 - (n_2 c_2 - s_2 h_2) \dot{\beta}_2 &= q_1 (u_1 + f_1), \\ j_y \dot{\alpha}_2 - (s_1^2 h_1 + c_2^2 h_2) \alpha_1 + (s_1 c_1 h_1 - s_2 c_2 h_2) \alpha_2 + \\ + (s_1 h_1 - c_1 n_1) \dot{\beta}_1 - (n_2 s_2 + c_2 h_2) \dot{\beta}_2 &= q_2 (u_2 + f_2). \end{aligned} \quad (12)$$

where $\beta_1(t)$, $\beta_2(t)$ are measured precession angles of gyros, $\alpha_1(t)$, $\alpha_2(t)$ are projections of platform absolute angular velocities on its axes, $u_1(t)$, $u_2(t)$ are moments of motors. Technological (assumed) parameters of the gyroplatform and the disturbance are: $p_1 = p_2 = 10^{-5} \cdot s^2$, $q_1 = q_2 = 10^{-5}$, $n_1 = n_2 = 4 \cdot 10^{-3}$, $h_1 = h_2 = 10^{-2}$, $j_x = 10^{-3} \cdot s^2$, $j_y = 2 \cdot 10^{-3} \cdot s^2$, $s_1 = \sin \delta_1$, $s_2 = \sin \delta_2$, $c_1 = \cos \delta_1$, $c_2 = \cos \delta_2$, $\delta_1 = -20^\circ$, $\delta_2 = 30^\circ$, $f_1 = 0$, $f_2 = 10 \sin 6.1t$.

The gyroplatform equations was converted to a state-space form. Matrices of this form are shown in Fig. 1, where parameters of the disturbance, the test signal and so on are shown too.

Gyroplatform transfer matrix corresponding these

parameters is

$$\begin{aligned} W(s) &= \\ &= \frac{1}{-1} \cdot \\ &\begin{pmatrix} (s+393.3)(s+370.8)(s+29.15)(s+12.41)s \\ 0.00342(s-2745)(s+390.6)(s+3.899) \\ 0.00866(s-560.5)(s+407.6)(s-23.07) \\ 0.0047(s+33.45)(s^2+732.8s+136400) \\ 0.0025(s+1724)(s+375.5)(s+31.01) \end{pmatrix}. \end{aligned} \quad (13)$$

In order to identified gyroplatform the following test signals are applied sequently to each its input.

$$\begin{aligned} u_1(t) &= \begin{bmatrix} u(t) \\ 0 \end{bmatrix} & 0 \leq t < 82, \\ u_2(t) &= \begin{bmatrix} 0 \\ u(t) \end{bmatrix} & 82 \leq t < 164. \end{aligned}$$

where

$$u(t) = 2 \sin t + 20 \sin 40t + 200 \sin 100t + 200 \sin 400t.$$

A result of identification by directive **M111** is the following transfer matrix

$$\begin{aligned} \widehat{W}(s) &= \\ &= \frac{1}{(s+376.4)(s+346.5)(s+29.53)(s+12.38)} \\ &\begin{pmatrix} (s+4.5 \cdot 10^{-4})(s+2.3 \cdot 10^{-4}) \\ 2.6 \cdot 10^{-3}(s-3402)(s+370.4)(s+3.921)(s+4.2 \cdot 10^{-4}) \\ 8.1 \cdot 10^{-3}(s-575.7)(s+380.4)(s-23.24)(s+2.8 \cdot 10^{-4}) \\ 4.7 \cdot 10^{-3}(s+33.21)(s+4.9 \cdot 10^{-4})(s^2+671.6s+123100) \\ 2.9 \cdot 10^{-3}(s+1392)(s+365.7)(s+31.13)(s+1.3 \cdot 10^{-4}) \end{pmatrix}. \end{aligned} \quad (14)$$

It is easily seen that the transfer matrices (13) and (14) are close.

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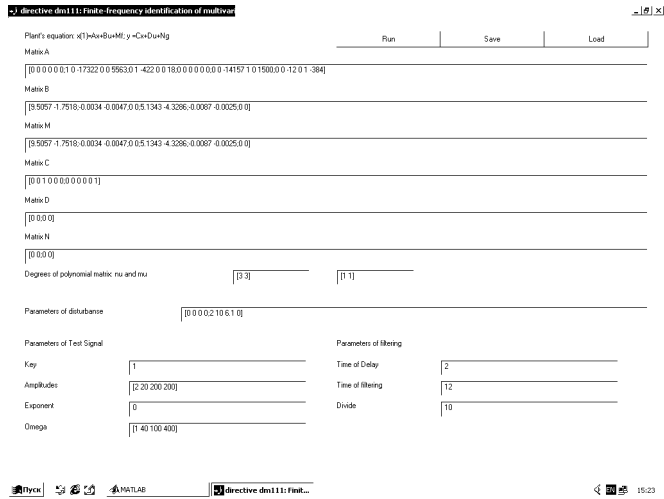


Fig. 1.