Preprints, IFAC Conference on Manufacturing Modelling,
Management and Control
June 28-30, 2016. Troyes, France

Parametric identification of nonlinear model for managed pressure drilling

Albert G. Alexandrov * **Vadim A. Alexandrov** ** **Dmitriy V. Shatov** ***

* *V.A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences (e-mail: alex7@ipu.ru).*
** *ADAPLAB LLC (e-mail: v.alexandrov@adaplab.ru)*
*** *V.A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences (e-mail: dvshatov@gmail.com)*

Abstract: Managed pressure drilling process is descried by nonlinear model. Nonlinear controller is proposed for this process. Model parameter is identified which is required to synthesize controller. Dynamic algorithm of frequency domain identification is used to estimate this parameter value.

Keywords: adaptive control, identification, nonlinear systems, nonlinear control, managed pressure drilling problem.

1. INTRODUCTION

Drilling fluid (mud) is pumped into the drill string during the drilling process. Hydrostatic pressure due to the weight of the mud and dynamic pressure from the pump both affect the drilling bit to overcome the rock resistance. The drilling mud from the drill string flows through the drilling bit to the annulus and rises to the surface taking out cuttings and cavings. The mud is recycled and returned back to the mud pit again.

The bottom hole pressure (BHP) must be controlled, let's denote this pressure p_b. BHP should be maintained within "drilling window". The drilling window limits are determined by geology of the well area and formation features. Actual pressure profile is almost always different from the theoretical one during real drilling, it can be substantially narrower or cause various other difficulties during drilling. The BHP must be rapidly changed in these cases.

Many technologies have been designed to improve the quality of the drilling process. Managed pressure drilling (MPD) is one of them. The main feature of this technology is sealing of the wellhead and installation of the choke valve to control the drilling mud flow through the annulus. There are a number of different approaches to the the choke valve control problem. Most of them are based on the are based on adaptive control laws. In (Mahdianfar, H. et al. (2013)) modified Kalman filter is proposed to be used for estimation of the BHP. The filter parameters are adapted to changes in the system. In (Nygaard, G. et al. (2006)) known scheme based on the model predictive control is applied. The paper considers using simplified low order model of the drilling process to control the bottom hole pressure in real time. In (Stamnes, Øy. N. et al. (2008); Li, Z. et al. (2012)) the problem of maintaining BHP within given limits is solved using an adaptive observer for estimating unknown parameters of the drilling process model. The L_1-adaptive control algorithm is built based on the previously mentioned process model. In (Zhou, J. et al. (2011)) adaptive observer is used, then the control law is designed in the form of a switch controller. Switching instants are determined by the inequalities, that include auxiliary parameters of the observer.

A new adaptation algorithm for choke valve control is proposed in this paper. The algorithm is a variant of the frequency adaptation method (Alexandrov, A. G. et al. (2002, 2006)). Distinctive feature is the use of non-linear control law, which requires parametric identification of the non-linear MPD system hydraulic model. Parametric identification is carried out using the dynamic algorithm of finite-frequency identification (Alexandrov, A. G. et al. (2009)), because the common finite-frequency identification algorithm makes obtaining of non-linear model parameter from frequency domain problematic.

2. MPD MODEL

Hydraulic drilling processes are described by partial differential nonstationary equations, their coefficients are changed as the depth of the well increases. These partial differential equations are usually replaced by a simple low order dynamic model (Stamnes, Øy. N. et al. (2008); Godhavn, J.-M. et al. (2011)). Fig. 1 shows the scheme of the MPD drilling system. The pump uploads mud from the pit into the drill string. Then the mud flows through the drilling bit through the annulus to the surface. The wellhead is hermetically sealed and all mud flows through the MPD choke valve which enables control of the pressure in the annulus.

The pressure dynamics in the volumes of the drill string and the annulus are described by equations based on the mass balance:

$$\frac{V_a}{\beta_a}\dot{p}_c = -\dot{V}_a + q_b + q_{res} - q_c, \qquad (1)$$

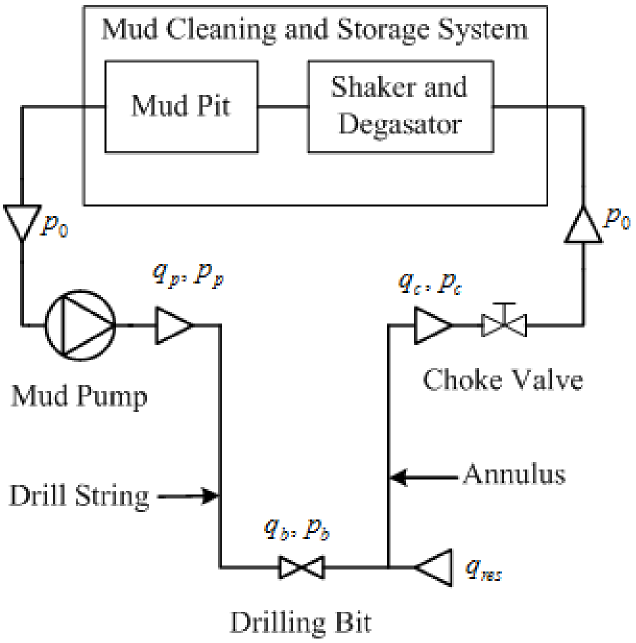

Fig. 1. The MPD drilling system

$$\frac{V_d}{\beta_d}\dot{p}_p = -q_p - q_b, \qquad (2)$$

where V_d is the drill string volume, β_d is the mud bulk modulus in the drill string, p_p is the pump pressure, q_p is the pump flow, q_b is the flow through the drilling bit, V_a is the annulus volume, β_a is the mud bulk modulus in the annulus, p_c is the pressure on the choke, q_{res} is the influx flow from the reservoir, q_c is the flow through the choke.

The drilling bit flow equation is based on the law of conservation of momentum:

$$\begin{aligned}[M_a + M_d]\dot{q}_b = p_p - p_c - F_d q_b^2 \\ -F_a(q_b + q_{res})^2 + (\rho_a - \rho_d) g h_b,\end{aligned} \qquad (3)$$

where M_a, M_d are mass coefficients of the annulus and the drill string, h_b is the depth of the drilling bit, ρ_a, ρ_d are average mud densities in the annulus and the drill string, F_a, F_d are friction coefficients of the annulus and the drill string, g is the gravity acceleration.

The equation of the flow through the choke depending on its position is the following:

$$q_c = k_c z_c \sqrt{\frac{2}{\rho_0}(p_c - p_0)} \qquad (4)$$

where k_c is the choke gain, z_c is the choke position expressed as opening/closing percentage (choke valve is closed when $z_c = 0$ and opened when $z_c = 100$), p_0 is the atmospheric pressure, ρ_0 is the mud density at atmospheric pressure.

The bottom hole pressure p_b is described by the following equation:

$$p_b = p_c + M_a \dot{q}_b + F_a(q_b + q_{res})^2 + \rho_a g h_b. \qquad (5)$$

Most papers consider the problem of maintaining constant bottom hole pressure p_b.

3. PROBLEM STATEMENT

The equation (5) is usually simplified by assuming that the BHP is described by the equation

$$p_b = p_c + a(h_b) \qquad (6)$$

where $a(h_b)$ is some known function that allows determining of the bottom hole pressure depending on the well depth. Under this assumption the problem is reduced to maintaining of given choke pressure p_c^*. The setpoint p_c^* is calculated based on the current value of $a(h_b)$ according to (6).

Therefore based on the aforementioned assumption instead of (1)-(3) a single equation of mass balance can be used to describe the process dynamics. The only equation is following:

$$\frac{V_a}{\beta_a}\dot{p}_c = f - q_c \qquad (7)$$

where $f = q_b + q_{res} - \dot{V}_a$ is external disturbance, which is assumed to be limited.

The function of the flow through the choke is described by equation (4). The differential equation describing the process dynamics is obtained by substituting (4) in (7):

$$\frac{V_a}{\beta_a}\dot{p}_c = f - k_c z_c \sqrt{\frac{2}{\rho_0}(p_c - p_0)}. \qquad (8)$$

Denoting $T = \dfrac{V_a}{\beta_a}$, $K_c = k_c\sqrt{\dfrac{2}{\rho_0}}$, $y = p_c$, $u = z_c$ the following differential equation for the MPD process dynamics is obtained:

$$T\dot{y} = f - K_c u \sqrt{y - p_0}. \qquad (9)$$

The problem of choke control is now to maintain the choke pressure at a given setpoint y^*.

4. CONTROL LAW SYNTHESIS

We need to design a controller that provides stability of the system and maintains the desired pressure y^*. Assume that the coefficients T and K_c are known.

Let's define a new variable

$$\tilde{u} = u\sqrt{y - p_0}.$$

The process then takes the following form:

$$T\dot{y} = f - K_c \tilde{u}. \qquad (10)$$

PI-controller can be used to control this process:

$$\dot{\tilde{u}}(t) = r_1 \dot{\varepsilon}(t) + r_0 \varepsilon(t) \qquad (11)$$

where $\varepsilon(t) = y^* - y(t)$.

Characteristic polynomial of the closed loop system can be obtained by differentiating (10) and substituting the control equation (11) into it:

$$T\ddot{y} = -K_c \dot{\tilde{u}} = -K_c[r_1 \dot{\varepsilon}(t) + r_0 \varepsilon(t)]. \qquad (12)$$

After applying Laplace transformation we obtain:

$$s^2 y = -\frac{K_c}{T}(r_1 s + r_0)\varepsilon.$$

Now the characteristic polynomial $D(s)$ of the system (10)-(11) has the following form:

$$D(s) = s^2 - \frac{K_c}{T}(r_1 s + r_0). \tag{13}$$

Let $r_1 = -\frac{T}{K_c}\widetilde{r}_1$ and $r_0 = -\frac{T}{K_c}\widetilde{r}_0$, then finally:

$$D(s) = s^2 + \widetilde{r}_1 s + \widetilde{r}_0. \tag{14}$$

It is sufficient to choose $\widetilde{r}_1 > 0$ and $\widetilde{r}_0 > 0$ to provide stability for the system (10)-(11). The control quality will depend on the particular choice of values $\widetilde{r}_1$ and $\widetilde{r}_0$. For example, to avoid overshoot, parameters $\widetilde{r}_1$ and $\widetilde{r}_0$ must be chosen so as $D(s)$ has real roots.

The control signal $u(t)$ in equation (9) is expressed in an explicit form. Therefore for practical application of the proposed approach we need to get a formula for its calculation. The expression for $\dot{\tilde{u}}$ is:

$$\dot{\tilde{u}} = \frac{\mathrm{d}(u\sqrt{y-p_0})}{\mathrm{d}t} = \dot{u}\sqrt{y-p_0} + \frac{\dot{y}}{2\sqrt{y-p_0}}u.$$

Then controller is described by the following equation

$$\dot{u}\sqrt{y-p_0} + \frac{\dot{y}}{2\sqrt{y-p_0}}u = r_1\dot{\varepsilon}(t) + r_0\varepsilon(t)$$

or, after transformation:

$$\dot{u} = -\frac{\dot{y}}{2(y-p_0)}u + \frac{1}{\sqrt{y-p_0}}(r_1\dot{\varepsilon}(t) + r_0\varepsilon(t)). \tag{15}$$

The equation (15) can be used to computing the control signal.

5. IDENTIFICATION OF MPD PARAMETERS

Let's now discard the assumption of known parameters K_c and T. We introduce the ratio

$$\alpha = -\frac{K_c}{T},$$

that is also unknown obviously.

Note that this ratio is only necessary for determination of the closed loop characteristic polynomial (13) parameters r_1 and r_0. Identification of the value of α is carried out using the dynamic finite-frequency identification algorithm (Alexandrov, A. G. et al. (2009)). During the identification procedure test harmonic signal is fed to the system as control signal:

$$u(t) = u_0 + \rho \sin\omega t, \tag{16}$$

where u_0, ρ and ω are chosen parameters of the test signal.

This test signal can be used to estimate the value of parameter α. Equation (9) is converted to difference equation using substitution $\dot{y}(t) = \frac{y(k) - y(k-1)}{h}$, where h is given sampling period. After division of both sides of the equation by $\frac{T}{h}$ one can obtain:

$$\begin{gathered} y(k) - y(k-1) = \frac{hf(k-1)}{T} \\ -\alpha h u(k-1)\sqrt{y(k-1) - p_0}, k = 1, 2, 3, 4, \dots, N \end{gathered} \tag{17}$$

where N is some given number.

The equation (17) is multiplied by modulating function $\sin\omega hk$ and summed for k:

$$\sum_{k=1}^{N}[y(k) - y(k-1)]\sin\omega hk = \sum_{k=1}^{N}\frac{hf(k-1)}{T}\sin\omega hk$$
$$-\alpha h\sum_{k=1}^{N}[u(k-1)\sqrt{y(k-1)-p_0}]\sin\omega hk,$$

The estimate $\hat{\alpha}$ can now be found:

$$\hat{\alpha} = -\frac{\sum\limits_{k=1}^{N}[y(k) - y(k-1)]\sin\omega hk}{h\sum\limits_{k=1}^{N}[u(k-1)\sqrt{y(k-1)-p_0}]\sin\omega hk}. \tag{18}$$

Equation (18) is valid because according to (Alexandrov, A. G. et al. (2009)) the following limit holds true

$$\frac{\sum\limits_{k=1}^{N}\frac{f(k-1)}{T}\sin\omega hk}{\sum\limits_{k=1}^{N}[u(k-1)\sqrt{y(k-1)-p_0}]\sin\omega hk} \to 0$$

when $N \to \infty$.

6. EXAMPLE

The system (9) with the following numerical parameters is considered:

$$\begin{gathered} T = 10; K_c = 0.4309; q_b = 285; \\ q_{res} = 30\mathrm{sign}[\sin(0.1)t]; \dot{V}_a = 0. \end{gathered} \tag{19}$$

Then the true value of the parameter α to be identified is $\alpha = -\frac{K_c}{T} = -0.0431$. Control signal lies within the range that corresponds to the choke valve opening/closing percentage, namely $u \in [0; 100]$.

The goal is to maintain pressure at the constant setpoint y^*. Two problems need to be solved: identification of parameter α with the proposed approach and synthesis of controller for estimate of the parameter.

6.1 Identification of parameter α

Identification of parameter α is carried out in an open loop. The test signal is:

$$u(t) = 50 + 5\sin(1.2t).$$

The offset value in the test signal is 50, it was chosen so that the pressure of the system (19) after transient is equal to the required setpoint $y^* = 190$.

Simulation was performed with the sampling period $h = 0.5$ seconds. The duration of the identification was 100 seconds ($N = 200$). The results of the simulation are presented in Fig. 2. The initial pressure is $y(0) = 150$.

Formula (18) gives the following estimate:

$$\hat{\alpha} = -0.0425.$$

One can see that this estimate is close to the true value of the parameter α.

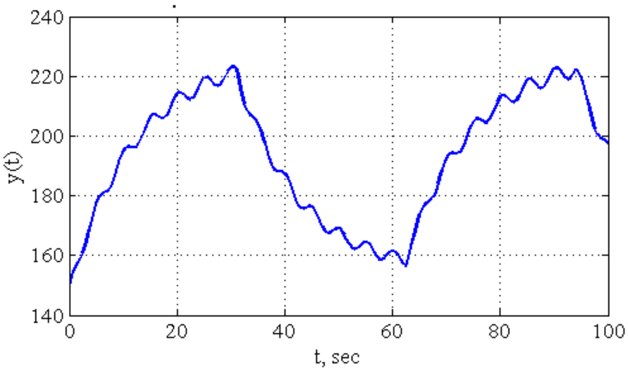

Fig. 2. Pressure during identification

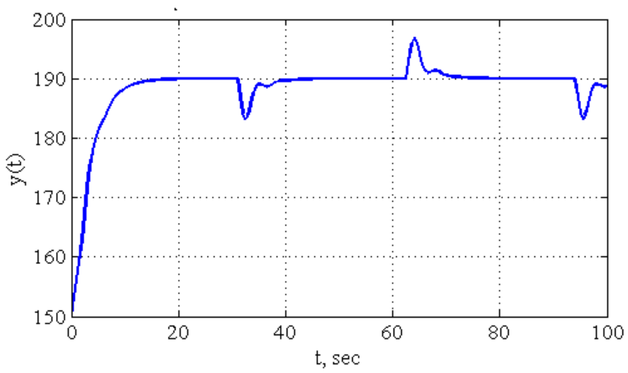

Fig. 3. Pressure in closed loop system

6.2 Controller synthesis

One must set the characteristic polynomial (13) of the closed loop system. Let $\widetilde{r}_1 = 0.6$ and $\widetilde{r}_0 = 0.5$, then for previously identified $\hat{\alpha} = -0.0425$ controller parameters (15) become the following

$$r_1 = -25.88;$$
$$r_0 = -7.06$$

Finally, the controller equation is obtained:

$$\hat{u} = -\frac{\dot{y}}{2y}u - \frac{1}{\sqrt{y}}(25.88\dot{\varepsilon} + 7.06\varepsilon). \tag{20}$$

The simulation results of the closed loop system are shown in Fig. 3. The parameters for the simulation were kept the same as for the identification experiment.

The open loop system operation with the same parameters and control value $u = 50$ is shown in Fig. 4 for comparison. One can clearly see that control quality in the closed loop system is noticeably improved.

7. CONSIDERATION OF CHOKE VALVE DYNAMICS

7.1 Control law synthesis

The choke valve drive has internal dynamics with time constant T_z. After taking that fact into account, instead of equation (9) one obtains the following system:

$$\begin{gathered} T\dot{y} = f - K_c u\sqrt{y - p_0} \\ T_z\dot{u} + u = u_c, \end{gathered} \tag{21}$$

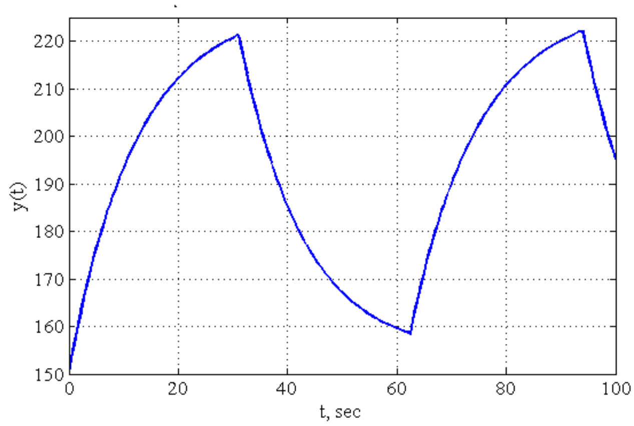

Fig. 4. Pressure in open loop system

where u_c is the choke valve drive input signal formed by the controller.

Let's consider the closed loop system (21) characteristic polynomial to synthesize the control law

$$\begin{aligned} T\ddot{y} = -K_c\left[\dot{u}\sqrt{y-p_0} + u\frac{\dot{y}}{2\sqrt{y-p_0}}\right] = \\ -K_c\left[\left(-\frac{1}{T_z}u + \frac{1}{T_z}u_c\right)\sqrt{y-p_0} + \frac{\dot{y}}{2\sqrt{y-p_0}}u\right] = \\ -K_c\left[\frac{\sqrt{y-p_0}}{T_z}u_c + \left(-\frac{\sqrt{y-p_0}}{T_z} + \frac{\dot{y}}{2\sqrt{y-p_0}}\right)u\right]. \end{aligned}$$

Finally, one obtains

$$\begin{aligned} \ddot{y} = \frac{\dot{f}}{T} - \frac{K_c}{T}\left[\frac{\sqrt{y-p_0}}{T_z}u_c\right. \\ \left. + \left(-\frac{\sqrt{y-p_0}}{T_z} + \frac{\dot{y}}{2\sqrt{y-p_0}}\right)u\right]. \end{aligned} \tag{22}$$

We use the previously described approach to control law synthesize. We need to find such a controller that transforms the closed loop system into a linear one, its polynomial will look similar to equation (12):

$$\ddot{y}(t) = -\frac{K_c}{T}[r_1\dot{\varepsilon}(t) + r_0\varepsilon(t)], \tag{23}$$

where $\varepsilon(t) = y^* - y(t)$.

The desired control law can be derived from the following equation

$$\frac{\sqrt{y-p_0}}{T_z}u_c + \left(-\frac{\sqrt{y-p_0}}{T_z} + \frac{\dot{y}}{2\sqrt{y-p_0}}\right)u = r_1\dot{\varepsilon} + r_0\varepsilon.$$

Finally, one gets the control law:

$$u_c = u - \frac{T_z\dot{y}}{2(y-p_0)} + \frac{T_z}{\sqrt{y-p_0}}(r_1\dot{\varepsilon} + r_0\varepsilon). \tag{24}$$

Substitution of (24) in (22) obviously leads to (23).

Let's introduce variables

$$\widetilde{r}_1 = -\frac{T}{K_c}r_1, \quad \widetilde{r}_0 = -\frac{T}{K_c}r_0 \tag{25}$$

then the characteristic polynomial of the closed loop system described by equations (21), (24) has the same form as (14). One can choose the controller parameters r_1 and r_0 based on the same previously described rules.

We consider that drive dynamics parameter T_z is known (from technical documentation or as a result of separate experiment). If T_z is known, then the control law (24) can be used to control the choke valve position, but as it follows from (25) to compute controller parameters $\widetilde{r}_1$ and $\widetilde{r}_0$ one needs to know the value of parameter $\alpha = -\dfrac{K_c}{T}$.

The estimate of α can be found using (18) again, but in this case the control signal u_c has the form (16), consequently the actual choke valve position u will be described by the following equation

$$u = u_0 + \rho_s \sin \omega t + \rho_c \cos \omega t,$$

where ρ_s, ρ_c depend on values of T_z, ρ and ω. Obviously, the harmonic component used in (18) passes through the choke valve drive, so estimate $\hat{\alpha}$ can be found using (18).

7.2 *Example*

Let's perform simulation of system (21) with parameter $T_z = 5$. Other parameters match those in (19). Let's assume parameter α is known for simplicity. Setpoint, initial pressure and control limits are kept the same:

$$y^* = 190, \; y(0) = 150, \; u, u_c \in [0; 100].$$

Initial position of the choke valve is set at $u(0) = 100$, the sampling period is $h = 0.1$.

The choke valve is controlled according to control law (24) using the following controller parameters (25):

$$\widetilde{r}_1 = -27.85, \quad \widetilde{r}_0 = -8.12,$$

or $r_1 = 0.8$ and $r_0 = 0.15$ respectively.

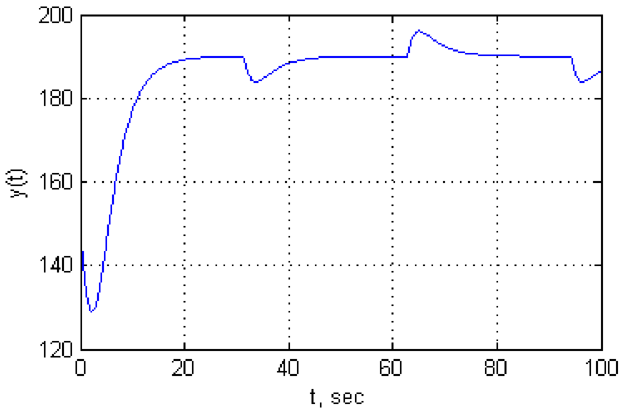

Fig. 5. Pressure in closed loop system with choke drive dynamics taking into account

Numerical implementation of the proposed control algorithm is complicated because it contains derivatives of signals $y(t)$ and $\varepsilon(t)$. There are several methods to numerically compute derivatives. In this example we use simple Euler approximation:

$$\dot{y}(t) = \frac{y(kh) - y[(k-1)h]}{h} \quad \dot{\varepsilon}(t) = \frac{\varepsilon(kh) - \varepsilon[(k-1)h]}{h}.$$

The benefit of this method is that it provides stability of the system for large enough values of the sampling periods h.

The choke valve pressure transient during numerical simulation of the system (21) with controller (24) is shown in the Fig. 5.

System (21) is more complex in comparison with equation (9), therefore the value α may be estimated with poorer accuracy. The proposed control law (24) is robust, because it provides stability of the system (21) even in situations when α is identified with large error. The Figures 6, 7 show pressure transient for two sets of controller parameters (25), when inaccurate estimate of the parameter α is found. Let's define true value α as α_T, pressure transient for $\alpha = 0.75\alpha_T$ is shown in the Fig. 6 and pressure transient for $\alpha = 1.25\alpha_T$ is shown in the Fig. 7. These Figures have no significant differences from the Fig. 3.

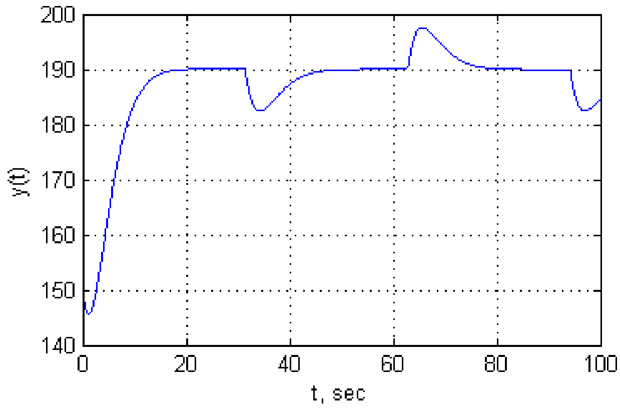

Fig. 6. Pressure in closed loop system with $\alpha = 0.75\alpha_T$

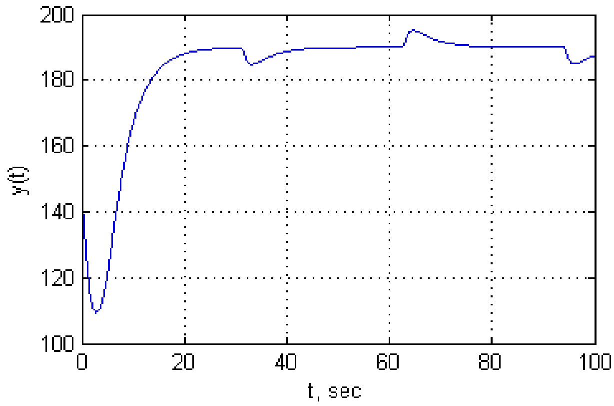

Fig. 7. Pressure in closed loop system with $\alpha = 1.25\alpha_T$

Numerical simulation of the system (21) closed with a typical PI-controller was performed to compare its performance against the proposed controller (24). The PI-controller is described by the following equation:

$$u_c = (1.5 + \frac{0.05}{s})\varepsilon.$$

Its parameters were chosen manually.

The Fig. 8 shows pressure transient with this PI-controller. One can observe significant overshoot and poor accuracy.

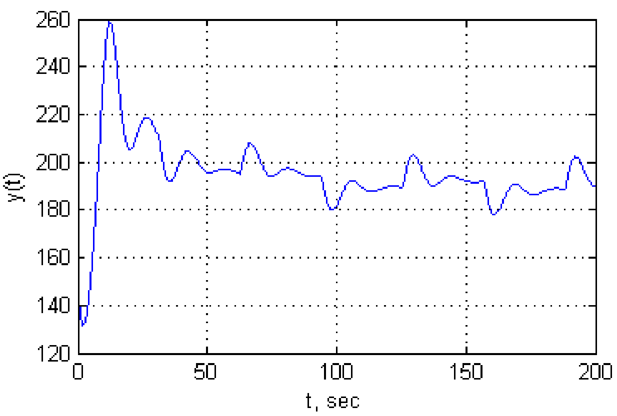

Fig. 8. Pressure in closed loop system with typical PI-regulator

8. CONCLUSIONS

Dynamic algorithm of finite-frequency identification is used to identify the parameter of managed pressure drilling process. The obtained estimate of the parameter is further used to synthesize control law. The synthesis procedure produces the controller, that compensates nonlinearity of the control system. Numerical simulations showed effectiveness of the proposed algorithms for parameter identification and controller synthesis.

REFERENCES

Alexandrov, A. G., Orlov, Yu. F. (2002). Frequency adaptive control of multivariable plants. *Preprints of the 15th Trienial World Congress of the IFAC*, Barcelona, Spain, (on CD-ROM, T-Th-M03, 3b).

Alexandrov, A. G., Orlov, Yu. F. (2006). A frequency adaptive control for multidimensional systems. *Automation and Remote Control*, Vol. 6, No 7, 1108-1122.

Alexandrov, A. G., Orlov, Yu. F. (2009). Finite-frequency identification: dynamical algorithm. *Control problems*, No 4, 2-8, (in Russian).

Godhavn, J.-M., Pavlov, A., Kaasa, G.-O., Rolland, N. L. (2011). Drilling seeking automatic control solutions. *Preprints of the 18th IFAC World Congress*, Milano, Italy. 10842-10850.

Li, Z., Hovakimyan, N., Kaasa, G.-O. (2012). Bottomhole pressure estimation and L_1 adaptive control in managed pressure drilling system. *Proceedings of the 2012 IFAC Workshop on Automatic Control in Offshore Oil and Gas Production*, Trondheim, Norwegian, 128-133.

Mahdianfar, H., Pavlov, A., Aamo, O. M. (2013). Joint unscented Kalman filter for state and parameter estimation in managed pressure drilling. *European Control Conference (ECC)*, Zürich, Switzerland, 1645-1650.

Nygaard, G., Nævdal, G. (2006). Nonlinear model predictive control scheme for stabilizing annulus pressure during oil well drilling. *Journal of Process Control*, Vol. 16, 719-732.

Stamnes, Øy. N., Zhou, J., Kaasa, G.-O., Aamo, O. M. (2008). Adaptive observer design for the bottomhole pressure of a managed pressure drilling dystem. *Proceedings of the 47th IEEE Conference on Decision and Control*, Cancun, Mexico, 2961-2966.

Zhou, J., Nygaard G. (2011). Automatic model-based control scheme for stabilizing pressure during dual-gradient drilling. *Journal of Process Control*, Vol. 21, Iss. 8, 1138-1147.