FREQUENCY ADAPTIVE CONTROL OF STABLE PLANT IN THE PRESENCE OF BOUNDED DISTURBANCE

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1. INTRODUCTION

Two branches of adaptive control theory may be extracted. They are distinguished by assumption about external disturbance.

The latter is absend [1] or a "white noise" [2] in the *first branch* which has large history connected, in particular, with the model reference adaptive systems and making use of the least squares techniques. The last survey of this branch is given in [3].

Since early 80's the *second branch* where disturbance is unknown-but-bounded is developed. A maximal amplitude of an unknown disturbance for which takes place the bounded processes in model reference adaptive systems has been obtained in [4]. The method of adaptive control based on the recurrent targeted inequalities has been supposed by [5] [6] for plant in the presence of unknown bounded disturbance. Least squares estimation algorithm with deadzone was used in [7] for solution of the same problem.

A number of adaptation algorithm originated from a notion of the frequency domain parameters [8] were proposed in [9]-[11]. These parameters are found by experiment in which a plant is excited by a test signal in form of a harmonics sum (with minimal quantity of harmonics: $p = n$, where n is a plant state space dimension) multiplied by $e^{\lambda t}$ ($\lambda > 0$) and plant output multiplied by $e^{-\lambda t}$. It allows on the one hand to develop frequency domain approach for an unstable plant and on the other hand to prove convergence to the true coefficients of plant or controller in the presence of any unknown-but-bounded disturbance. However, a growing test signal creates [12] a problem for a bounded input of plant.

In this paper a stable plant excited by the test signal under $\lambda = 0$ is considered. An adaptation algorithm and conditions of its converge is obtained.

2. PROBLEM STATEMENT

Consider a completely controllable and asymptotically stable plant described by the following differential equation

$$y^{(n)}+d_{n-1}y^{(n-1)}+\cdots+d_0y=k_\gamma u^{(\gamma)}+\cdots+k_0u+f, \quad t\geq t_0, \quad (1)$$

where $y(t)$ is a measured output, $u(t)$ is an input to be controlled, $y^{(i)}$, $u^{(j)}$ $\left(i = \overline{1,n},\ j = \overline{1,\gamma}\right)$ are the derivatives of these functions, $f(t)$ is an unknown-but-bounded disturbance. The coefficients d_i and k_j $\left(i = \overline{0,n-1},\ j = \overline{0,\gamma}\right)$ are some unknown numbers, n is known, $\gamma < n-1$. The initial conditions $y^{(i)}(t_0)$ $\left(i = \overline{0,n-1}\right)$ and disturbance $f(t)$ satisfy the following inequalities

$$|y^{(i)}(t_0)| \leq \varepsilon_i^0, \quad i = \overline{0,n-1} \quad (2)$$

$$|f(t)| \leq f^*, \quad (3)$$

in which ε_i^0 $\left(i = \overline{0,n-1}\right)$ and f^* are numbers.

The plant input is formed by the following controller with the piecewise-constant coefficients

$$g_{n-1}^{[i]}u^{(n-1)}+\cdots+g_0^{[i]}u=r_{n-1}^{[i]}y^{(n-1)}+\cdots+r_0^{[i]}y+v^{[i]}, \quad t_{i-1}\leq t<t_i, \quad u^{(p)}(t_{i-1})=0 \quad i = \overline{1,N} \quad (4)$$

where i $\left(i = \overline{1,N}\right)$ is an adaptation interval number, the time of ending adaptation intervals t_i $\left(i = \overline{1,N}\right)$ as well as numbers $g_k^{[i]}$, $r_k^{[i]}$ $\left(k = \overline{0,n-1},\ i = \overline{1,N}\right)$ are found in an adaptation process,

$$v^{[i]}(t)=\sum_{k=1}^{q}\rho_k \sin\omega_k(t-t_{i-1}), \quad t_{i-1}\leq t<t_i \quad i = \overline{1,N} \quad (5)$$

are the test signals with the test frequencies $\omega_k > 0$ and amplitudes ρ_k $\left(k=\overline{1,q}\right)$.

On some of adaptation intervals, in particular, under $i=1$, the differential equation (4) has a simple view

$$u = v^{[i]} \quad i \in \overline{1,N} \\ \text{(that is } g_k^{[i]}{=}r_k^{[i]}{=}0 \ \left(k{=}\overline{1,n{-}1}\right),\ g_0^{[i]}{=}1,\ r_0^{[i]}{=}0). \tag{6}$$

In those cases test signals (5) contain n harmonics ($q=n$) and in remaining cases $q=2n$.

Systems (1), (6) and (1), (4) will reffered to as the open and closed-loop systems correspondly. The plant inputs of these systems are denoted as y_{op} and y_{cl}. Under $v^{[i]}=0$ $(i \in \overline{1,N})$ they are denoted as y^f_{op} and y^f_{cl}.

The amplitudes of test signal (5) have to meet the following demands of "small excitation"

$$\max_{t_0 \le t < t_N} \left| y_{op} - y^f_{op} \right| \le \bar{\varepsilon}, \quad \max_{t_0 \le t < t_N} \left| y_{cl} - y^f_{cl} \right| \le \bar{\varepsilon}, \tag{7}$$

where $\bar{\varepsilon}$ is a given number. These demands may include the analogous inequalities for plant inputs.

In this paper ways of amplitudes tuning provided these demands are omitted and it is assumed that amplitudes ρ_k and frequencies ω_k $\left(k=\overline{1,n}\right)$ are specified.

In the moment time t_N when adaptation process is ended controller (4) has the following form

$$g_{n-1}u^{(n-1)}+\cdots+g_0u=r_{n-1}y^{(n-1)}+\cdots+r_0y, \quad t \ge t_N, \tag{8}$$

where $g_k = g_k^{[N]}$, $r_k = r_k^{[N]}$ $\left(k=\overline{0,n-1}\right)$.

The characteristic polynomial of system (1), (8) is

$$\varphi(s) = d(s)g(s) - k(s)r(s) = \varphi_{2n-1}s^{2n-1}+\cdots+\varphi_0 \tag{9}$$

where $d(s) = s^n + d_{n-1}s^{n-1} + \cdots + d_0$, $g(s) = g_{n-1}s^{n-1} + \cdots + g_0$, $k(s) = k_\gamma s^\gamma + \cdots + k_0$, $r(s) = r_{n-1}s^{n-1} + \cdots + r_0$.

Problem 2.1 Find a adaptation algorithm of coefficients of controller (4) such that the characteristic polynomial of system (1), (8) and a specified Hurwitz's polynomial

$$\psi(s) = \psi_{2n-1}s^{2n-1} + \cdots + \psi_0 \tag{10}$$

meet the following demands

$$|\psi_i - \varphi_i| \le \varepsilon_i^\psi \quad i=\overline{0,2n-1} \tag{11}$$

where ε_i^ψ $\left(i=\overline{0,2n-1}\right)$ are given numbers ■.

3. PLANT IDENTIFICATION

3.1 Frequency domain parameters (FDP) of plant.

On the first interval the plant (1) is excited by the test signal (6). That is

$$u(t) = v^{[1]}(t) = \sum_{k=1}^{n} \rho_k \sin \omega_k (t - t_0). \tag{12}$$

A set of $2n$ numbers α_k and β_k $\left(k=\overline{1,n}\right)$ is called [8] the *frequency domain parameters* (FDP) of plant. It connects with its transfer function $w(s) = k(s)/d(s)$ as

$$\alpha_k = \operatorname{Re} w(j\omega_k), \quad \beta_k = \operatorname{Im} w(j\omega_k) \quad k=\overline{1,n}. \tag{13}$$

If the plant output $y(t) = y_{op}(t)$ is applied to Fourier's filter then their outputs give the following FDP estimates

$$\begin{aligned} \hat{\alpha}_k = \alpha_k(\tau) &= \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \sin \omega_k (t - t_0)\, dt \\ & \qquad\qquad\qquad\qquad k=\overline{1,n}, \\ \hat{\beta}_k = \beta_k(\tau) &= \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \cos \omega_k (t - t_0)\, dt \end{aligned} \tag{14}$$

where τ is a filtering time (duration of an interval), t_F is a filtering start time, the test frequencies ω_k $\left(k=\overline{1,n}\right)$ are multiple some basic frequency ω_b: $\omega_k = \tilde{q}_k \omega_b$ where $\tilde{q}_k$ $\left(k=\overline{1,n}\right)$ are positive integers, filtering time τ is multiple a period $T_b = \frac{2\pi}{\omega_b}$.

3.2 Frequency equations of identification.

Consider identity

$$w(j\omega) = k(j\omega)/d(j\omega). \tag{15}$$

It follows the equations

$$k(j\omega_k) - (\alpha_k + j\beta_k)\bar{d}(j\omega_k) = (\alpha_k + j\beta_k)(j\omega_k)^n \quad k=\overline{1,n} \tag{16}$$

where $\bar{d}(s) = d(s) - s = d_{n-1}s^{n-1} + \cdots + d_0$.

If plant (1) is completely controllable and the frequencies ω_k $\left(k=\overline{1,n}\right)$ are positive and different then system (16) has the unique solution k_i, d_i $\left(i=\overline{0,n-1}\right)$, $(k_{n-1} = \cdots = k_{\gamma+1} = 0)$. [8]

Rewriting system (16) in more detail form and substituting FDP of their estimates the following system referred to as *the frequency equations of identification* [10] is obtained

$$\sum_{i=0}^{n-2} \hat{k}_i (j\omega_k)^i - (\hat{\alpha}_k + j\hat{\beta}_k) \sum_{i=0}^{n-1} \hat{d}_i (j\omega_k)^i = \\ = (\hat{\alpha}_k + j\hat{\beta}_k)(j\omega_k)^n \quad k = \overline{1,n} \tag{17}$$

3.3 Algorithm.

The first interval duration $\tau = qT_b$ is found by the following necessary conditions of identification convergence

$$\begin{array}{ll} |d_i(qT_b) - d_i[(q+1)T_b)]| \le \varepsilon_i^d & i = \overline{0, n-1} \\ |k_i(qT_b) - k_i[(q+1)T_b)]| \le \varepsilon_i^k & q = 1, 2, \ldots \end{array} \tag{18}$$

where ε_i^d , ε_i^k $\left(i = \overline{0, n-1}\right)$ are given numbers.

Algorithm 3.1 (finite-frequency identification):

- apply output of plant (1) exited by the test signal (12) to input of Fourier's filter (14),
- measure Fourier's filter output in the time moments $\tau = qT_b$ ($q = 1, 2, \ldots$),
- solve for each time moment τ the frequency equations (17), where $\hat{\alpha}_k = \alpha_k(qT_b)$, $\hat{\beta}_k = \beta_k(qT_b)$ $\left(k = \overline{1,n}\right)$, and find the estimates $\hat{d}_i(qT_b)$, $\hat{k}_i(qT_b)$ $\left(i = \overline{0, n-1}\right)$.
- examine the necessary conditions (18) for each q till these conditions hold for some $q = q_1$.

■

4. ITERATIVE CLOSED LOOP IDENTIFICATION AND CONTROLLER REDESIGN

Algorithm 3.1 results in the plant coefficients estimates $\hat{d}_i = d_i(q_1 T_b)$ and $\hat{k}_j = k_j(q_1 Tb)$ $\left(i = \overline{0, n–1}, j = \overline{0, n–2}\right)$ Forming the polynomials $\hat{d}(s) = s^n + \hat{d}_{n-1} s^{n-1} + \cdots + \hat{d}_0$, $\hat{k}(s) = \hat{k}_{n-2} s^{n-2} + \cdots + \hat{k}_0$ and soling the following Bezout-Identity

$$\hat{d}(s)\hat{g}(s) - \hat{k}(s)\hat{r}(s) = \psi(s) \tag{19}$$

the coefficients of controller

$$g^{[2]}(s)u = r^{[2]}(s)y + v^{[2]} \tag{20}$$

(where $g^{[2]}(s) = \hat{g}(s)$, $r^{[2]}(s) = \hat{r}(s)$) are found. Consider system (1), (20). It may be rewritten as

$$\varphi^{[2]}(s)y = k(s)v^{[2]} + g^{[2]}f \tag{21}$$

where

$$\varphi^{[2]} = d(s)g^{[2]}(s) - k(s)r^{[2]}(s) \tag{22}$$

Making use of algorithm 3.1 for "a plant" (21) the following estimates are derived

$$\begin{array}{l} \hat{\varphi}_i^{[2]} = \varphi_i^{[2]}(q_2 T_b) \\ \hat{k}_j^{[2]} = k_j^{[2]}(q_2 T_b) \end{array} \quad i = \overline{0, 2n-1} \quad j = \overline{0, n-2} \tag{23}$$

(In considered case the second sum of frequency equations (17) has upper limit equaled $2n-1$ and symbols $\hat{d}_i$ of this sum are substituted by $\hat{\varphi}_i^{[2]}$ $\left(i = \overline{0, 2n-1}\right)$. In addition, in expressions for Fourier's filter (14) the plant output $y(t) = y_{cl}(t)$).

If the requirements (11):

$$|\psi_i - \hat{\varphi}_i^{[2]}| \le \varepsilon_i^{\psi} \quad i = \overline{0, 2n-1} \tag{24}$$

hold then $N = 2$ and the seeked polynomials of controller (8) are

$$g(s) = g^{[2]}(s), \quad r(s) = r^{[2]}(s) \tag{25}$$

If the contrary is the case the numbers of the right parts of inequalities (18) are decreased and algorithm 3.1 continues to operate until a new value $q = q_2' > q_2$ and so on. If it does not lead to reaching the targeted conditions (24) then it means that identification accuracy obtained on the first interval is not sufficiently. Then the FDP estimates of system (1), (21) are used for an improvement of the first intervals results.

In fact, introduce a set of the following $2n$ numbers for each interval (they are called *FDP of a closed-loop system* for the i-th interval)

$$\begin{array}{l} \nu_k^{[i]} = \mathrm{Re}\, w_{cl}^{[i]}(j\omega_k) \\ \mu_k^{[i]} = \mathrm{Im}\, w_{cl}^{[i]}(j\omega_k) \end{array} \quad k = \overline{1,n} \quad i = \overline{1,N} \tag{26}$$

where

$$w_{cl}^{[i]}(s) = \frac{k(s)}{\varphi^{[i]}(s)} \quad i = \overline{1,N} \tag{27}$$

It is easily shown that the plant and closed–loop system FDP are linked as

$$\alpha_k + j\beta_k = \frac{\nu_k^{[i]} + j\mu_k^{[i]}}{(\nu_k^{[i]} + j\mu_k^{[i]})w_c^{[i]}(j\omega_k) + w_l^{[i]}(j\omega_k)} \\ k = \overline{1,n} \quad i = \overline{1,N} \tag{28}$$

where $w_l^{[i]}(s) = \dfrac{r^{[i]}(s)}{g^{[i]}(s)}$, $w_l^{[i]}(s) = \dfrac{1}{g^{[i]}(s)}$.

Substituting $\nu_k^{[2]}$ and $\mu_k^{[2]}$ $\left(k = \overline{1,n}\right)$ by their estimates $\nu_k^{[2]}(qT_b)$ and $\mu_k^{[2]}(qT_b)$, ($k = \overline{1,n}$, $q = q_2, \ldots$) (obtained on the outputs of Fourier's filter (14) under

$y(t) = y_{cl}(t)$) the FDP estimates of plant $\alpha_k(qT_b)$, $\beta_k(qT_b)$ ($k = \overline{1,n}$, $q = q_2, \dots$) are found from expression (28).

Using algorithm 3.1 the new estimates $\hat{d}_i$, $\hat{k}_j$ $(i = \overline{0,n-1},\ j = \overline{0,n-2})$ are found. Solving Bezout-Identity (19) the coefficients of controller

$$g^{[3]}(s)u = r^{[3]}(s)y + v^{[3]}$$

are derived and so on.

5. ADAPTATION PROCESS CONVERGENCE

Introduce the functions $l_k^\alpha(\tau)$, $l_k^\beta(\tau)$ $(k = \overline{1,n})$ that are the output of Fourier's filter (14) excited by plant output $y(t) = y_{op}^f(t)$ (it means that in equation (1) $u(t) = 0$).

In the paper it is proved that frequency adaptation process is convergent and therefore the aim (11) are reached if there exists a time moment τ^* such that the following conditions hold

$$|l_k^\alpha(\tau)| \le \varepsilon_k^\alpha, \quad |l_k^\beta(\tau)| \le \varepsilon_k^\beta, \quad \tau \ge \tau^* \quad k = \overline{1,n} \quad (29)$$

where ε_k^α and ε_k^β $(k = \overline{1,n})$ are sufficiently small given numbers.

Condition (29) may be examined by experiment.

6. EXAMPLE

In the paper the example illustrated efficiency of proposed method of adaptation is given.

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