FREQUENCY ADAPTIVE CONTROL OF MULTIVARIABLE PLANTS

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Abstract: Proposed is a technique of adaptive control for a multivariable plant in the presence of bounded polyharmonic disturbance with infinity number of harmonics and with the unknown amplitudes and frequencies. The control objective is to provide the prescribed tolerances on forced oscillation of the plant and controller outputs. The adaptation process is based on the finite-frequency identification of the plant and closed-loop system. Adaptation terminates when the coefficients of the identified plant-controller system is close to those of the identified closed-loop system. Convergence conditions of the adaptation procedure are derived. They can be tested experimentally.

Keywords: Multivariable plant, adaptive control, unknown-but-bounded disturbance, test signal.

1. INTRODUCTION

In adaptive control it may be extracted two directions that are differed by assumptions on external disturbance.

In the framework of the first direction the external disturbance is absent (Anderson *et al.*, 1986) or it is a “white-noise” (Iserman, 1981). The direction has large history connected, in particular, with the model reference adaptive systems and the least squares techniques. The last survey of this direction is given in (Landau, 1997).

Since early 80’s the second direction where disturbance is unknown-but-bounded time function is being developed: method of the recurrent targeted inequalities (Yakubovich, 1988), least squares estimation algorithm with dead zone (Zhao and Lozano, 1993), frequency adaptive control (Alexandrov, 1998), and so on. The control aim in these techniques of second direction is described by a polynomial with prescribed poles placement.

For many practical cases the control aim is the prescribed tolerances on the deviation of the plant output from zero. Technique of controller design for this case has been proposed in (Alexandrov and Chestnov, 1997, 1998). In this case the plant coefficients are known and the disturbance is a bounded polyharmonic function with known number of harmonics of unknown amplitudes and frequencies.

In the present paper this technique is being developed for a plant with unknown coefficients and the disturbance with infinity number of harmonics.

Unlike the above mentioned papers of the second direction, where single-input-single-output plant is considered, this paper deals with multivariable plant. It is well known (Guidorzi, 1975; Gauthier and Landau, 1978) that identification problem of multivariable plant has not unique solution. In the paper (Orlov, 2000) a structure, for which finite-frequency identification gives unique solution, has been obtained and it is used below to design an algorithm of adaptive control.

The paper is organized as follows. In section 2 a problem of adaptive control design is formulated and its solution for known coefficients of plant is given in section 3. In sections 4 and 5 adaptation algorithm, which is based on finite-frequency identification of the plant and closed-loop system, is derived. Conditions of adaptation convergence is studied in section 6.

2. PROBLEM STATEMENT

Consider a linear time-invariant system described by the following equations

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{u}+\boldsymbol{w}), \quad \boldsymbol{y} = \boldsymbol{z} = \boldsymbol{C}\boldsymbol{x}, \quad t \geq t_0, \tag{1}$$

$$\dot{\boldsymbol{x}}_c = \boldsymbol{A}_c\boldsymbol{x}_c + \boldsymbol{B}_c\boldsymbol{y}, \quad \boldsymbol{u} = \boldsymbol{C}_c\boldsymbol{x}_c, \tag{2}$$

where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$ is the state vector of plant (1), $\boldsymbol{x}_c(t) \in \boldsymbol{R}^n$ is the state vector of controller (2), $\boldsymbol{u}(t) \in \boldsymbol{R}^m$ is the input to be controlled, $\boldsymbol{y}(t) \in \boldsymbol{R}^r$ is the measurable output, $\boldsymbol{z}(t) \in \boldsymbol{R}^r$ is the controlled output, $\boldsymbol{w}(t) \in \boldsymbol{R}^m$ is the external unmeasurable disturbance, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{A}_c$, $\boldsymbol{B}_c$, $\boldsymbol{C}_c$ are unknown constant matrices. The pair $(\boldsymbol{A}, \boldsymbol{B})$ is controllable and pair $(\boldsymbol{A}, \boldsymbol{C})$ is observable. The disturbance components are bounded polyharmonic functions

$$w_j(t) = \sum_{k=1}^{\infty} w_{jk} \sin(\omega_k t + \psi_{jk}), \quad j = \overline{1, m}, \tag{3}$$

where the frequencies ω_k and the phases ψ_{jk} $\left(j = \overline{1, m}, \ k = \overline{1, \infty}\right)$ are unknown numbers and amplitudes w_{jk} satisfy the conditions

$$\sum_{k=1}^{\infty} w_{jk}^2 \leq w_j^{*2}, \quad j = \overline{1, m}, \tag{4}$$

where w_j^* $\left(j = \overline{1, m}\right)$ are given numbers.

As $t \to \infty$ forced oscillations of outputs of plant and controller are described by expressions

$$z_i(t) = \sum_{k=1}^{\infty} a_i(\omega_k) \sin[\omega_k t + \phi_i(\omega_k)], \quad i = \overline{1, r}, \tag{5}$$

$$u_j(t) = \sum_{k=1}^{\infty} b_j(\omega_k) \sin[\omega_k t + \theta_j(\omega_k)], \quad j = \overline{1, m}. \tag{6}$$

The matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ of plant (1) have the following property: there exist matrices $\boldsymbol{A}_c$, $\boldsymbol{B}_c$ and $\boldsymbol{C}_c$ of controller (2) such that the amplitudes of forced oscillations of the plant and controller outputs satisfy the following conditions

$$\sum_{k=1}^{\infty} a_i^2(\omega_k) \leq a_i^{*2}, \ i = \overline{1, r}, \ \sum_{k=1}^{\infty} b_j^2(\omega_k) \leq b_j^{*2}, \ j = \overline{1, m}, \tag{7}$$

where a_i^* and b_j^* $\left(i = \overline{1, r}, \ \ j = \overline{1, m}\right)$ are given numbers.

Since the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are unknown, adaptive control must be used. In this case, controller (2) is described by the following equations with piecewise-constant coefficients

$$\begin{gathered}\dot{\boldsymbol{x}}_c = \boldsymbol{A}_c^{[\kappa]}\boldsymbol{x}_c + \boldsymbol{B}_c^{[\kappa]}\boldsymbol{y} + \boldsymbol{L}\boldsymbol{v}^{[\kappa]}, \quad \boldsymbol{u} = \boldsymbol{C}_c^{[\kappa]}\boldsymbol{x}_c, \\ t_{\kappa-1} \leq t < t_\kappa \quad \kappa = \overline{1, N},\end{gathered} \tag{8}$$

where κ $\left(\kappa = \overline{1, N}\right)$ is the number of adaptation interval, t_κ is the termination time of the κ-th interval; t_κ as well as the number N and the matrices $\boldsymbol{A}_c^{[\kappa]}$, $\boldsymbol{B}_c^{[\kappa]}$ and $\boldsymbol{C}_c^{[\kappa]}$ are found during the adaptation process; $\boldsymbol{L}$ is a given matrix; $\boldsymbol{v}^{[\kappa]}(t) \in \boldsymbol{R}^m$ is a test signal, whose components are defined below.

The adaptation process is terminated (at moment t_N) and the controller is described by the equations (2) where $\boldsymbol{A}_c = \boldsymbol{A}_c^{[N]}$, $\boldsymbol{B}_c = \boldsymbol{B}_c^{[N]}$ and $\boldsymbol{C}_c = \boldsymbol{C}_c^{[N]}$.

Problem 1 Find an adaptation algorithm for the coefficients of controller (8) such that the system (1), (2) meet the requirements (7) for steady-state amplitudes of forced oscillations.

3. CONTROL OF AMPLITUDES OF FORCED OSCILLATIONS FOR KNOWN PLANT

If the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ of plant (1) are known, the matrices of controller (2), which solves problem 1 is found from the expressions

$$\begin{gathered}\boldsymbol{A}_c = \boldsymbol{A} - \boldsymbol{B}(\boldsymbol{R}^{-1} - \gamma^{-2}\boldsymbol{Q}_1)\boldsymbol{B}^T\boldsymbol{P} - \boldsymbol{K}_f\boldsymbol{C}, \ \boldsymbol{B}_c = \boldsymbol{K}_f, \\ \boldsymbol{C}_c = -\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}, \ \boldsymbol{K}_f = (\boldsymbol{E}_n - \gamma^{-2}\boldsymbol{Y}\boldsymbol{P})^{-1}\boldsymbol{Y}\boldsymbol{C}^T,\end{gathered} \tag{9}$$

where the square non-negative matrices $\boldsymbol{P}$ and $\boldsymbol{Y}$ of size $n \times n$ are the solution of the Riccati equations

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}(\boldsymbol{R}^{-1} - \gamma^{-2}\boldsymbol{Q}_1)\boldsymbol{B}^T\boldsymbol{P} = -\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}, \tag{10}$$

$$\boldsymbol{A}\boldsymbol{Y} + \boldsymbol{Y}\boldsymbol{A}^T - \boldsymbol{Y}\boldsymbol{C}^T(\boldsymbol{E}_r - \gamma^{-2}\boldsymbol{Q})\boldsymbol{C}\boldsymbol{Y} = -\boldsymbol{B}\boldsymbol{Q}_1\boldsymbol{B}^T, \tag{11}$$

where number γ satisfies the condition

$$\lambda_{\max}(\boldsymbol{P}\boldsymbol{Y}) < \gamma^2, \tag{12}$$

and $\lambda_{\max}(\boldsymbol{M})$ is the maximal eigenvalue of the non-negative matrix $\boldsymbol{M}$. For $\boldsymbol{Q} = \boldsymbol{E}_r$ and $\boldsymbol{R} = \boldsymbol{Q}_1 = \boldsymbol{E}_m$ ($\boldsymbol{E}_\bullet$ is an identity matrix of an appropriate size) equations (10) and (11) coincide with the equations of H_∞-suboptimal control (Doyle, *et al.* 1989) (under the condition that $\boldsymbol{B}_1 = \boldsymbol{B}_2 = \boldsymbol{B}$ and $\boldsymbol{C}_1 = \boldsymbol{C}_2 = \boldsymbol{C}$).

Let $\boldsymbol{Q} = \mathrm{diag}[q_1, \ldots, q_r]$, $\boldsymbol{R} = \mathrm{diag}[r_1, \ldots, r_m]$, and $\boldsymbol{Q}_1 = \boldsymbol{E}_m$.

Assertion 1 If the elements of diagonal matrices $\boldsymbol{Q}$ *and* $\boldsymbol{R}$ *satisfy the inequalities*

$$q_i \geq \frac{1}{a_i^{*2}} \sum_{k=1}^{m} w_k^{*2}, \ i=\overline{1,r}, \quad r_j \geq \frac{1}{b_j^{*2}} \sum_{k=1}^{m} w_k^{*2}, \ j=\overline{1,m}, \tag{13}$$

the steady-state amplitudes of forced oscillations of system (1), (2) with matrices (9-11) satisfy the inequality

$$\sum_{i=1}^{r} \frac{1}{a_i^{*2}} \sum_{k=1}^{\infty} a_i^2(\omega_k) + \sum_{j=1}^{m} \frac{1}{b_j^{*2}} \sum_{k=1}^{\infty} b_j^2(\omega_k) \leq \gamma^{*2}, \tag{14}$$

where γ^* *is the least value of* γ*, such that* $\boldsymbol{P}$ *and* $\boldsymbol{Y}$ *are non-negative matrices and condition (12) holds.*

From inequality (14) it follows that, if $\gamma^* \leq 1$, system (1), (2) with coefficients (9) satisfy requirement (7) on amplitudes of forced oscillations.

The assertion is a generalization of the theorem 5 in (Alexandrov and Chestnov, 1998, part II) whose proof based on a lemma in (Alexandrov and Chestnov, 1998, part I). If in proof of lemma a frequencies number (p) is equal to infinity ($p = \infty$) and Cauchy-Bunyakovski's inequality is not used then proof the assertion is a repetition of proof of theorem 5.

4. THE FIRST INTERVAL OF ADAPTATION

4.1 Frequency Domain Parameters of Plant.

Let, for simplicity, plant (1) be asymptotically stable and its observability indices ν_i $(i = \overline{1,r})$ are known. To find matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ the plant is excited by the following test signals

$$\boldsymbol{u}_j(t) = \sum_{k=1}^{n} \rho_{jk}^u \sin \omega_k^u t \cdot \boldsymbol{e}_j, \qquad j = \overline{1,m}, \tag{15}$$
$$t_0 + (j-1)\tau^{[1]} \leq t < t_0 + j\tau^{[1]},$$

where ρ_{jk}^u $(j = \overline{1,m}, \ k = \overline{1,n})$ is the specified amplitude of the k-th harmonics of test signal for the j-th experiment, ω_k^u $(k = \overline{1,n})$ is the specified test frequency [$\omega_k^u \neq 0$ $(k = \overline{1,n})$ and $\omega_i^u \neq \omega_j^u$ $(i \neq j)$], $\boldsymbol{e}_j$ is the j-th column of a identity matrix $\boldsymbol{E}_m$, $\tau^{[1]}$ is a duration of the j-th experiment, $\tau^{[1]}$ is a given number such that $t_0 + m\tau^{[1]} = t_1$. This number may be found by experiment on the base of necessary conditions of identification convergence.

Plant outputs $\boldsymbol{y}_j(t)$ $(j = \overline{1,m})$ are applied to inputs of the Fourier's filter, whose outputs give the estimates

$$\hat{\alpha}_{ijk} = \frac{2}{\rho_{jk}^u \tau^{[1]}} \int_{t_0+(j-1)\tau^{[1]}}^{t_0+j\tau^{[1]}} y_{ji}(t) \sin \omega_k^u (t - t_0)\, dt,$$

$$\hat{\beta}_{ijk} = \frac{2}{\rho_{jk}^u \tau^{[1]}} \int_{t_0+(j-1)\tau^{[1]}}^{t_0+j\tau^{[1]}} y_{ji}(t) \cos \omega_k^u (t - t_0)\, dt, \tag{16}$$
$$i = \overline{1,r}, \quad j = \overline{1,m}, \quad k = \overline{1,n},$$

for elements α_{ijk} and β_{ijk} of matrices $\boldsymbol{A}_k = \mathrm{Re}\,\boldsymbol{W}(\jmath\omega_k^u)$ and $\boldsymbol{B}_k = \mathrm{Im}\,\boldsymbol{W}(\jmath\omega_k^u)$ $(k = \overline{1,n})$ which are named *Frequency Domain Parameters (FDP)* (Alexandrov, 1989) of plant (1), where $\boldsymbol{W}(s) = \boldsymbol{C}(\boldsymbol{E}s - \boldsymbol{A})^{-1}\boldsymbol{B}$ is its transfer matrix.

4.2 Plant Identification.

The estimates of matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ of plant (1) are searched in Luenberger's canonical form and these are denoted $\hat{\boldsymbol{A}}^K$, $\hat{\boldsymbol{B}}^K$ and $\hat{\boldsymbol{C}}^K$. Blocks $\hat{\boldsymbol{A}}_{ij}^K$ and $\hat{\boldsymbol{c}}_{ij}^K$ $(i, j = \overline{1,r})$ of matrices $\hat{\boldsymbol{A}}^K$ and $\hat{\boldsymbol{C}}^K$ have the following structure

$$\begin{aligned}
\hat{\boldsymbol{A}}_{ii}^K &= \begin{pmatrix} 0 & 0 & \cdots & 0 & -\hat{d}_{ii}^{(0)} \\ 1 & 0 & \cdots & 0 & -\hat{d}_{ii}^{(1)} \\ 0 & 1 & \cdots & 0 & -\hat{d}_{ii}^{(2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\hat{d}_{ii}^{(\nu_i - 1)} \end{pmatrix}, \\
\hat{\boldsymbol{A}}_{i\neq j}^K &= \begin{pmatrix} 0 & 0 & \cdots & 0 & -\hat{d}_{ij}^{(0)} \\ 0 & 0 & \cdots & 0 & -\hat{d}_{ij}^{(1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -\hat{d}_{ij}^{(\nu_{ij} - 1)} \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}; \\
\hat{\boldsymbol{c}}_{ii}^K &= \begin{pmatrix} 0 & \cdots & 0 & 1 \end{pmatrix}, \ \nu_{ij} = \min(\nu_i, \nu_j), \\
\hat{\boldsymbol{c}}_{i>j}^K &= \begin{pmatrix} 0 & \cdots & 0 & -\hat{d}_{ij} \end{pmatrix}, \ \hat{\boldsymbol{c}}_{i<j}^K = \boldsymbol{0}.
\end{aligned} \tag{17}$$

To find the coefficients of matrices $\boldsymbol{A}^{[1]} = \hat{\boldsymbol{A}}^K$, $\boldsymbol{B}^{[1]} = \hat{\boldsymbol{B}}^K$ and $\boldsymbol{C}^{[1]} = \hat{\boldsymbol{C}}^K$ the following frequency equations of identification (Orlov, 2000)

$$\sum_{i=1}^{m} \sum_{j=0}^{\nu_k - 1} \boldsymbol{i}_i^{(j)} \hat{g}_{ki}^{(j)} + \sum_{i=1}^{r} \sum_{j=0}^{\breve{\nu}_{ki} - 1} \hat{\boldsymbol{h}}_i^{(j)} \hat{f}_{ki}^{(j)} = -\hat{\boldsymbol{h}}_k^{(\nu_k)}, \ k = \overline{1,r} \tag{18}$$

are solved and coefficients $\hat{d}_{ij}^{(k)}$ and $\hat{d}_{ij}$ are calculated as

$$\hat{d}_{ij}^{(k)} = \hat{f}_{ij}^{(k)} - \sum_{l=j+1}^{r} \hat{f}_{il}^{(k)} \hat{d}_{lj} \tag{19}$$
$$k = \overline{0, \nu_{ij} - 1} \quad i = \overline{1,r} \quad j = \overline{1,r}.$$

$$\hat{d}_{ij} - \sum_{k=j+1}^{i-1} \hat{d}_{ik} \cdot \hat{f}_{kj}^{(\breve{\nu}_{kj} - 1)} + \hat{f}_{ij}^{(\breve{\nu}_{ij} - 1)} = 0 \tag{20}$$
$$i = \overline{j+1, r} \quad j = \overline{1, r-2},$$

Here $\boldsymbol{i}_i^{(j)}=[\,\mathrm{Re}\, s_1^j \boldsymbol{e}_i \;\; \mathrm{Im}\, s_1^j \boldsymbol{e}_i \;\cdots\; \mathrm{Re}\, s_n^j \boldsymbol{e}_i \;\; \mathrm{Im}\, s_n^j \boldsymbol{e}_i\,]^T$ and $\hat{\boldsymbol{h}}_i^{(j)}=[\,\mathrm{Re}[s_1^j \hat{\boldsymbol{w}}_i(s_1)] \;\; \mathrm{Im}[s_1^j \hat{\boldsymbol{w}}_i(s_1)] \;\cdots\; \mathrm{Re}[s_n^j \hat{\boldsymbol{w}}_i(s_n)] \;\; \mathrm{Im}[s_n^j \hat{\boldsymbol{w}}_i(s_n)]\,]^T$, $\boldsymbol{e}_i$ and $\hat{\boldsymbol{w}}_i(s_k)$ are the i-th row of a matrices $\boldsymbol{E}_m$ and $\widehat{\boldsymbol{W}}(s_k) = \hat{\boldsymbol{A}}_k + \jmath\hat{\boldsymbol{B}}_k$ respectively, $s_k = \jmath\omega_k^u \;\; (k=\overline{1,n})$, $\breve{\nu}_{k<i} = \nu_{ki}$ and $\breve{\nu}_{k>i} = \min(\nu_k+1, \nu_i)$, $\hat{g}_{ki}^{(j)}$ are coefficients of matrix $\hat{\boldsymbol{B}}^K$. For convenience, equations (18) are derived in appendix.

4.3 Hypothetical model of closed-loop system.

Matrices $\boldsymbol{A}_c^{[2]}$, $\boldsymbol{B}_c^{[2]}$ and $\boldsymbol{C}_c^{[2]}$ of controller (8) for the second interval of adaptation are found by formula (9) after solution of Riccati equations (10) and (11) in which matrices $\boldsymbol{A}=\boldsymbol{A}^{[1]}$, $\boldsymbol{B}=\boldsymbol{B}^{[1]}$ and $\boldsymbol{C}=\boldsymbol{C}^{[1]}$, the components of matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are determined from inequalities (13), $\boldsymbol{Q}_1 = \boldsymbol{E}_m$ and $\gamma=\gamma^*$.

The plant and controller

$$\dot{\boldsymbol{x}}=\boldsymbol{A}^{[1]}\boldsymbol{x}+\boldsymbol{B}^{[1]}(\boldsymbol{u}+\boldsymbol{w}), \quad \boldsymbol{y}=\boldsymbol{z}=\boldsymbol{C}^{[1]}\boldsymbol{x}, \quad t\geq t_1, \tag{21}$$

$$\dot{\boldsymbol{x}}_c = \boldsymbol{A}_c^{[2]}\boldsymbol{x}_c + \boldsymbol{B}_c^{[2]}\boldsymbol{y} + \boldsymbol{L}\boldsymbol{v}^{[2]}, \quad \boldsymbol{u} = \boldsymbol{C}_c^{[2]}\boldsymbol{x}_c \tag{22}$$

are named the *hypothetical closed-loop system.* In Luenberger's canonic form this system has the following view

$$\dot{\bar{\boldsymbol{x}}} = \bar{\boldsymbol{A}}\bar{\boldsymbol{x}} + \bar{\boldsymbol{L}}\boldsymbol{v}^{[2]} + \bar{\boldsymbol{B}}\boldsymbol{w}, \quad \boldsymbol{y} = \bar{\boldsymbol{C}}\bar{\boldsymbol{x}}, \tag{23}$$

whose blocks $\bar{\boldsymbol{A}}_{ij}$ and $\bar{\boldsymbol{c}}_{ij}$ $(i=\overline{1,r}, \;\; j=\overline{1,r})$ have view (17) where $\hat{d}_{ij}^{(k)}$, $\hat{d}_{ij}$, ν_{ij} and ν_i are substituted by $\bar{d}_{ij}^{(k)}$, $\bar{d}_{ij}$, $\bar{\nu}_{ij}$ and $\bar{\nu}_i$; $\bar{\nu}_i$ are indices of an observability of the hypothetical system $\left(\sum_{i=1}^r \bar{\nu}_i = 2n\right)$.

5. THE SECOND INTERVAL OF ADAPTATION

5.1 The Frequency Domain Parameters of Closed-loop System.

Let system (1), (22) be excited by m-vectors of test signals

$$\boldsymbol{v}_j^{[2]}(t) = \sum_{k=1}^{2n} \rho_{jk}^v \sin\omega_k^v t \cdot \boldsymbol{e}_j, \qquad j=\overline{1,m}, \tag{24}$$
$$t_1 + (j-1)\tau^{[2]} \leq t < t_1 + j\tau^{[2]},$$

where ρ_{jk}^v $(j=\overline{1,m})$ is amplitude and ω_k^v is frequency of the k-th harmonic $(k=\overline{1,2n})$ of test signals $[\,\omega_k^v \neq 0 \;\; (k=\overline{1,2n})$ and $\omega_i^v \neq \omega_j^v \;\; (i\neq j)\,]$, $t_1 + m\tau^{[2]} = t_2$. Duration of each experiment

$$\tau^{[2]} = \tau^{[1]} + K, \tag{25}$$

where K is a given positive number.

Components of vectors $\boldsymbol{y}_j(t)$ $(j=\overline{1,m})$ are applied to inputs of the Fourier's filter whose outputs give the estimates

$$\hat{\nu}_{ijk} = \frac{2}{\rho_{jk}^v \tau^{[2]}} \int\limits_{t_1+(j-1)\tau^{[2]}}^{t_1+j\tau^{[2]}} y_{ji}(t) \sin\omega_k^v (t-t_1)\, dt,$$
$$\hat{\mu}_{ijk} = \frac{2}{\rho_{jk}^v \tau^{[2]}} \int\limits_{t_1+(j-1)\tau^{[2]}}^{t_1+j\tau^{[2]}} y_{ji}(t) \cos\omega_k^v (t-t_1)\, dt, \tag{26}$$
$$i=\overline{1,r}, \quad j=\overline{1,m}, \quad k=\overline{1,2n},$$

for elements ν_{jik} and μ_{jik} of matrices $\boldsymbol{\mathcal{V}}_k = \mathrm{Re}\,\widetilde{W}(\jmath\omega_k^v)$ and $\boldsymbol{\mathcal{M}}_k = \mathrm{Im}\,\widetilde{W}(\jmath\omega_k^v)$ $(k=\overline{1,2n})$ *of Frequency Domain Parameters (FDP)* of the closed-loop system. Here

$$\widetilde{\boldsymbol{W}}(s) = [\boldsymbol{E}_r - \boldsymbol{W}(s)\boldsymbol{W}_c(s)]^{-1}\boldsymbol{W}(s)\boldsymbol{W}_v(s), \tag{27}$$

where
$$\boldsymbol{W}_c(s) = \boldsymbol{C}_c^{[2]}(\boldsymbol{E}s - \boldsymbol{A}_c^{[2]})^{-1}\boldsymbol{B}_c^{[2]},$$
$$\boldsymbol{W}_v(s) = \boldsymbol{C}_c^{[2]}(\boldsymbol{E}s - \boldsymbol{A}_c^{[2]})^{-1}\boldsymbol{L}.$$

5.2 Closed-loop System Identification.

Solve the following system of frequency equations

$$\sum_{i=1}^{m}\sum_{j=0}^{\bar{\nu}_k-1} \tilde{\boldsymbol{i}}_i^{(j)} \hat{\tilde{l}}_{ki}^{(j)} + \sum_{i=1}^{r}\sum_{j=0}^{\breve{\nu}_{ki}-1} \hat{\tilde{\boldsymbol{h}}}_i^{(j)} \hat{\tilde{f}}_{ki}^{(j)} = -\hat{\tilde{\boldsymbol{h}}}_k^{(\bar{\nu}_k)}, \quad k=\overline{1,r}, \tag{28}$$

where $\tilde{\boldsymbol{i}}_i^{(j)} = [\;\mathrm{Re}\,\tilde{s}_1^j \boldsymbol{e}_i \;\; \mathrm{Im}\,\tilde{s}_1^j \boldsymbol{e}_i \;\cdots\; \mathrm{Re}\,\tilde{s}_{2n}^j \boldsymbol{e}_i \;\; \mathrm{Im}\,\tilde{s}_{2n}^j \boldsymbol{e}_i\;]^T$ and $\hat{\tilde{\boldsymbol{h}}}_i^{(j)} = [\;\mathrm{Re}[\tilde{s}_1^j \hat{\tilde{\boldsymbol{w}}}_i(\tilde{s}_1)] \;\; \mathrm{Im}[\tilde{s}_1^j \cdot \hat{\tilde{\boldsymbol{w}}}_i(\tilde{s}_1)] \;\cdots\; \mathrm{Re}[\tilde{s}_{2n}^j \hat{\tilde{\boldsymbol{w}}}_i(\tilde{s}_{2n})] \;\; \mathrm{Im}[\tilde{s}_{2n}^j \hat{\tilde{\boldsymbol{w}}}_i(\tilde{s}_{2n})]\;]^T$, $\boldsymbol{e}_i$ and $\hat{\tilde{\boldsymbol{w}}}_i(\tilde{s}_k)$ are the i-th row of matrices $\boldsymbol{E}_m$ and $\widehat{\widetilde{\boldsymbol{W}}}(\tilde{s}_k) = \widehat{\boldsymbol{\mathcal{V}}}_k + \jmath\widehat{\boldsymbol{\mathcal{M}}}_k$ respectively, $\tilde{s}_k = \jmath\omega_k^v$ $(k=\overline{1,2n})$, $\breve{\nu}_{k<i} = \bar{\nu}_{ki}$ and $\breve{\nu}_{k>i} = \min(\bar{\nu}_k+1, \bar{\nu}_i)$. It gives the coefficient estimates $\hat{\tilde{d}}_{ij}^{(k)}$ $(k=\overline{0,\bar{\nu}_{ij}-1}, \;\; i=\overline{1,r}, \;\; j=\overline{1,r})$ and $\hat{\tilde{l}}_{ij}^{(k)}$ $(k=\overline{0,\bar{\nu}_i-1}, \;\; i=\overline{1,r}, \;\; j=\overline{1,m})$ of the closed-loop system in the Luenberger's canonic form

$$\dot{\tilde{\boldsymbol{x}}} = \hat{\tilde{\boldsymbol{A}}}\tilde{\boldsymbol{x}} + \hat{\tilde{\boldsymbol{L}}}\boldsymbol{v}^{[2]} + \hat{\tilde{\boldsymbol{B}}}\boldsymbol{w}, \quad \boldsymbol{y} = \hat{\tilde{\boldsymbol{C}}}\tilde{\boldsymbol{x}}, \tag{29}$$

whose blocks $\hat{\tilde{\boldsymbol{A}}}_{ij}$ and $\hat{\tilde{\boldsymbol{c}}}_{ij}$ $(i=\overline{1,r}, \;\; j=\overline{1,r})$ have view (17) where $\hat{d}_{ij}^{(k)}$, $\hat{d}_{ij}$, ν_{ij} and ν_i are substituted by $\hat{\tilde{d}}_{ij}^{(k)}$, $\hat{\tilde{d}}_{ij}$, $\bar{\nu}_{ij}$ and $\bar{\nu}_i$, where $\hat{\tilde{d}}_{ij}^{(k)}$ and $\hat{\tilde{d}}_{ij}$ are found on the base of expressions which is analogous (20) and (19).

5.3 Conditions of adaptation completion.

Compare the hypothetical and identified systems (23) and (29) and examine the following inequalities

$$\bar{d}_{ij}^{(k)} \div \hat{\tilde{d}}_{ij}^{(k)} \leq \varepsilon_d, \quad k=\overline{0,\bar{\nu}_{ij}-1}, \quad i=\overline{1,r}, \quad j=\overline{1,r},$$
$$\bar{l}_{ij}^{(k)} \div \hat{\tilde{l}}_{ij}^{(k)} \leq \varepsilon_l, \quad k=\overline{0,\bar{\nu}_i-1}, \quad i=\overline{1,r}, \quad j=\overline{1,m}, \tag{30}$$
$$\bar{d}_{ij} \div \hat{\tilde{d}}_{ij} \leq \varepsilon_d, \quad i=\overline{j+1,r-1} \quad j=\overline{1,r-1},$$

where ε_d and ε_l are given numbers, symbol $\div$ means as follows $a \div b = |a-b|/|b|$ if $b \neq 0$ or $a \div b = |a|$ if $b = 0$.

If these inequalities are fulfilled then adaptation is ended and therefore $N = 2$ and matrices of controller (2) are: $\boldsymbol{A}_c = \boldsymbol{A}_c^{[2]}$, $\boldsymbol{B}_c = \boldsymbol{B}_c^{[2]}$ and $\boldsymbol{C}_c = \boldsymbol{C}_c^{[2]}$.

If the contrary is the case (which means that identification accuracy which is obtained on the first interval of adaptation is not sufficiently) two situations are possible: *a)* system (1), (22) is stable, *b)* this system is unstable. Consider each of these situations.

In the case a) matrices $\widehat{\boldsymbol{\mathcal{V}}}_k$ and $\widehat{\boldsymbol{\mathcal{M}}}_k$ of the closed-loop FDP estimates are used for improvement matrices $\hat{\boldsymbol{\mathcal{A}}}_k$ and $\hat{\mathcal{B}}_k$ $(k=\overline{1,n})$ of the plant FDP estimates. To this effect the following almost obvious relation serves

$$\boldsymbol{\mathcal{A}}_k + \jmath \boldsymbol{\mathcal{B}}_k = [\boldsymbol{\mathcal{V}}_k + \jmath \boldsymbol{\mathcal{M}}_k] \cdot \cdot \{\boldsymbol{W}_c(\tilde{s}_k)[\boldsymbol{\mathcal{V}}_k + \jmath \boldsymbol{\mathcal{M}}_k] + \boldsymbol{W}_v(\tilde{s}_k)\}^{-1} \quad k=\overline{1,n}. \tag{31}$$

Replacing in expression (31) the matrices $\boldsymbol{\mathcal{V}}_k$ and $\boldsymbol{\mathcal{M}}_k$ by their estimates, the new matrices $\hat{\boldsymbol{\mathcal{A}}}_k$ and $\hat{\boldsymbol{\mathcal{B}}}_k$ $(k=\overline{1,n})$ are calculated and matrices $\boldsymbol{A}^{[2]}$, $\boldsymbol{B}^{[2]}$ and $\boldsymbol{C}^{[2]}$ are found as a solution of frequency equations (18). Then the Riccati equations (10) and (11) are solved and matrices $\boldsymbol{A}_c^{[3]}$, $\boldsymbol{B}_c^{[3]}$ and $\boldsymbol{C}_c^{[3]}$ are calculated and so on.

In the case b) it need disconnect controller (22) and on the third interval the plant (1) is excited by test signals (15). However, duration of each test $\tau^{[3]}$ has to be more then the duration of the first interval and that is why

$$\tau^{[3]} = \tau^{[2]} + K. \tag{32}$$

Under this condition the plant (1) is identified, matrices $\boldsymbol{A}^{[3]}$, $\boldsymbol{B}^{[3]}$ and $\boldsymbol{C}^{[3]}$ are found and so on.

6. ADAPTATION PROCESS CONVERGENCE

Introduce the filterableness functions (Alexandrov, 1998)

$$\ell^{\alpha}_{ijk}(\tau) = \frac{2}{\rho_{jk}\tau} \int_{t_0+(j-1)\tau}^{t_0+j\tau} \bar{y}_{ji}(t) \sin\omega_k(t-t_0)\, dt,$$
$$\ell^{\beta}_{ijk}(\tau) = \frac{2}{\rho_{jk}\tau} \int_{t_0+(j-1)\tau}^{t_0+j\tau} \bar{y}_{ji}(t) \cos\omega_k(t-t_0)\, dt, \tag{33}$$
$$i=\overline{1,r}, \quad j=\overline{1,m}, \quad k=\overline{1,\theta n},$$

for multivariable plants. These are Fourier's filter outputs, whose the inputs are the "natural" outputs of plant (1), when $\boldsymbol{u}=\boldsymbol{0}$ (or system (1), (8), when $\boldsymbol{v}^{[\kappa]} = \boldsymbol{0}$); $\rho_{jk} = \rho^u_{jk}$, $\omega_k = \omega^u_k$, $\theta = 1$ (or $\rho_{jk} = \rho^v_{jk}$, $\omega_k = \omega^v_k$, $\theta = 2$).

A disturbance $\boldsymbol{w}(t)$ is named FF-filterable if there exists time of filtering τ^* such that the following conditions hold

$$\frac{|\ell^{\alpha}_{ijk}(\tau)|}{|\alpha_{ijk}(\tau)|} \leq \varepsilon^{\alpha}_k, \quad \frac{|\ell^{\beta}_{ijk}(\tau)|}{|\beta_{ijk}(\tau)|} \leq \varepsilon^{\beta}_k, \quad i=\overline{1,r}, \quad j=\overline{1,m}, \quad k=\overline{1,\theta n}, \quad \tau \geq \tau^*, \tag{34}$$

where ε^{α}_k and ε^{β}_k $(k=\overline{1,\theta n})$ are given numbers. Disturbance $\boldsymbol{w}(t)$ is strong FF-filterable when

$$\lim_{\tau\to\infty} \ell^{\alpha}_{ijk}(\tau) = \lim_{\tau\to\infty} \ell^{\beta}_{ijk}(\tau) = 0, \quad i=\overline{1,r}, \quad j=\overline{1,m}, \quad k=\overline{1,\theta n}. \tag{35}$$

If disturbance is FF-filterable then errors $\Delta\alpha_{ijk} = \alpha_{ijk} - \hat{\alpha}_{ijk}$ and $\Delta\beta_{ijk} = \beta_{ijk} - \hat{\beta}_{ijk}$ $(i=\overline{1,r},$ $j=\overline{1,m},$ $k=\overline{1,\theta n})$ satisfy the following inequalities

$$\frac{|\Delta\alpha_{ijk}(\tau)|}{|\alpha_{ijk}(\tau)|} \leq \varepsilon^{\alpha}_k, \quad \frac{|\Delta\beta_{ijk}(\tau)|}{|\beta_{ijk}(\tau)|} \leq \varepsilon^{\beta}_k, \quad i=\overline{1,r}, \quad j=\overline{1,m}, \quad k=\overline{1,\theta n}, \quad \tau \geq \tau^{\star}, \tag{36}$$

and for the strong FF-filterable disturbance

$$\lim_{\tau\to\infty} \Delta\alpha_{ijk}(\tau) = \lim_{\tau\to\infty} \Delta\beta_{ijk}(\tau) = 0 \quad i=\overline{1,r}, \quad j=\overline{1,m}, \quad k=\overline{1,\theta n}. \tag{37}$$

For errors $\Delta\nu_{ijk}$ and $\Delta\mu_{ijk}$ the expression analogous (36) and (37) takes place.

It is easly examined that if frequencies of a disturbance and test signals do not coincide:

$$\omega_k \neq \omega^u_i, \quad \omega_k \neq \omega^v_j \quad i=\overline{1,n}, \quad j=\overline{1,2n}, \quad k=\overline{1,\infty}, \tag{38}$$

then $\boldsymbol{w}(t)$ is strong FF-filterable.

Adaptation process converges if a time $\bar{\tau} > \tau^{\star}$ is reachable. From expression (25) and (32) for determination of test duration it follows that for any given value K there always exists number N such that any $\bar{\tau}$ is reached.

It is almost obvious as follows

Assertion 2 If disturbance $\boldsymbol{w}(t)$ is strong FF-filterable and

$$\tau^{[\kappa]} = \tau^{[\kappa-1]} + K, \quad \kappa = \overline{1,N}, \tag{39}$$

then adaptation process converges and requirements (7) hold, if the disturbance is FF-filterable then fulfilment of (7) depends on numbers ε^{α}_k and ε^{β}_k $(k=\overline{1,\theta n})$ and on analogous numbers for closed-loop system.

Remark 1 The "natural" outputs of the plant and system $\bar{y}_{ji}(t)$, $(i=\overline{1,r},\ \ j=\overline{1,m})$ use for choice of amplitudes of test signal from the following conditions (Alexandrov, 1998) of "small excitation"

$$\bar{y}_{ji}(t) \div y_{ji}(t) \leq \bar{\varepsilon}, \quad i=\overline{1,r}, \quad j=\overline{1,m},$$

where $\bar{\varepsilon}$ is a given number.

7. CONCLUSION

In this paper a new technique of adaptive control for a multivariable plant in the presence of the bounded polyharmonic disturbance (3) is proposed. The adaptive control is provided the requirements (7) to accuracy.

The technique is based on an experimental determination of the plant and closed-loop system FDP excited by "sufficiently small" test signals.

It consists of intervals on which the plant or closed-loop system are identified. Adaptation process is stopped when requirements (30) to nearness of the hypothetical and identified closed-loop system are fulfilled. Convergence of adaptation is proved.

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APPENDIX

System (18) follows from input-output description of polynomial matrices $\boldsymbol{W}(s)=\boldsymbol{F}^{-1}(s)\boldsymbol{G}(s)$ that may be rewritten as

$$\begin{aligned}&\left(\boldsymbol{G}^{(0)T}+\boldsymbol{G}^{(1)T}s+\cdots+\boldsymbol{G}^{(\gamma)T}s^{\gamma}\right)-\\&-\boldsymbol{W}^T(s)\left(\boldsymbol{F}^{(0)T}+\boldsymbol{F}^{(1)T}s+\cdots+\boldsymbol{F}^{(\eta)T}s^{\eta}\right)=0.\end{aligned} \tag{A.1}$$

Substituting in (A.1) $s=s_k=\jmath\omega_k$, $\boldsymbol{W}(s_k)=\boldsymbol{W}_k=\boldsymbol{A}_k+\jmath\boldsymbol{B}_k$ $(k=\overline{1,n})$, it is easily obtained the following system

$$\begin{aligned}&\sum_{j=0}^{\gamma}\operatorname{Re}s_k^j\boldsymbol{G}^{(j)T}-\sum_{j=0}^{\eta}\left[\operatorname{Re}s_k^j\boldsymbol{A}_k^T-\operatorname{Im}s_k^j\boldsymbol{B}_k^T\right]\boldsymbol{F}^{(j)T}=0\\&\qquad\qquad\qquad\qquad k=\overline{1,n}.\\&\sum_{j=0}^{\gamma}\operatorname{Im}s_k^j\boldsymbol{G}^{(j)T}-\sum_{j=0}^{\eta}\left[\operatorname{Im}s_k^j\boldsymbol{A}_k^T+\operatorname{Re}s_k^j\boldsymbol{B}_k^T\right]\boldsymbol{F}^{(j)T}=0\end{aligned}$$

The system has a infinity set of solutions: $\boldsymbol{F}^{(j)}=\left[\!\left[f_{ik}^{(j)*}\right]\!\right]\in\boldsymbol{R}^{r\times r}$ $(i=\overline{0,\eta})$ and $\boldsymbol{G}^{(j)}=\left[\!\left[g_{ik}^{(j)*}\right]\!\right]\in\boldsymbol{R}^{r\times m}$ $(i=\overline{0,\gamma})$. It is proved (Orlov, 2000) that solution of this system is unique if coefficients of polynomial matrices $\boldsymbol{F}(s)$ and $\boldsymbol{G}(s)$ is searched in the following form

$$\begin{aligned}f_{ii}(s)&=f_{ii}^{(0)}+f_{ii}^{(1)}s+\cdots+f_{ii}^{(\nu_i-1)}s^{\nu_i-1}+s^{\nu_i}\\f_{i\neq j}(s)&=f_{ij}^{(0)}+f_{ij}^{(1)}s+\cdots+f_{ij}^{(\nu_{ij}-1)}s^{\bar{\nu}_{ij}-1}\\g_{ik}(s)&=g_{ik}^{(0)}+g_{ik}^{(1)}s+\cdots+g_{ik}^{(\nu_i-1)}s^{\nu_i-1}\\i&=\overline{1,r},\quad j=\overline{1,r},\quad k=\overline{1,m}.\end{aligned} \tag{A.2}$$