Proceedings of the Tenth IASTED International Conference
CONTROL AND APPLICATIONS (CA 2008)
May 26-28, 2008 Quebec City, Quebec, Canada

FREQUENCY ADAPTIVE CONTROLLER: EXPERIMENTAL INVESTIGATIONS

*Albert Alexandrov & **Dmitry Khomutov
Institute of Control Sciences RAS
65 Profsojuznaja street, Moscow 117997, Russia
*alex7@ipu.ru, **dmitry.khomutov@gmail.com

ABSTRACT
This paper presents the results of experimental analysis of a frequency adaptive controller ChAR-22 in the feedback loop aimed at achieving a desired tolerance level for the plant output. The plant has some unknown parameters; furthermore, an external disturbance affects the plant. A physical plant model (PPM) was used in the experiments as a real plant. The PPM is implemented as an electronic device. Putting the CHAR-22 controller in the feedback loop reduces the system output more than by a factor of ten as compared to the output of the open-loop system.

KEY WORDS
Adaptive control, frequency identification, software, experimental investigation

1. Introduction

In adaptive control it may be extracted two directions that are differed by assumptions on external disturbance. In the framework of the first direction the external disturbance is absent [1] or it is a “white-noise” [2]. The direction has large history connected, in particular, with the model reference adaptive systems and the least squares techniques. The last survey of this direction is given in [3]. Since the early 80’s the second direction where disturbance is unknown-but-bounded time function is being developed: method of the recurrent targeted inequalities [4], least squares estimation algorithm with dead zone [5], frequency adaptive control [6] and so on. In this paper, experimental results of the frequency adaptive control method are given.

An adaptive control algorithm proposed in paper [7] produces the controller that provides specified tolerance for the plant output. In paper [8] algorithms of testing signal self-tunings is presented. The controller ChAR-21d, containing almost all of these algorithms, is described in paper [9].

In this paper the results of the experimental investigations with ChAR-22 are presented. The frequency adaptive controller ChAR-22 is represented as a code in the C language for IBM-compatible computers similarly to what was done for the ChAR-21d controller. Due to the specific identification algorithms exploited here (see [10]), the total time of the adaptation process in the ChAR-22 is decreased as compared to the one reported in [9].

2. A problem statement

Consider a closed-loop system with the plant which is shown in Figure 1. The plant is described by following difference equation:

$$\begin{aligned} &y(n+i)+d_{n-1}y(n+i-1)+\ldots+d_0y(i)= \\ &=k_{n-1}u(n+i-1)+\ldots+k_0u(i)+f(i), \end{aligned} \qquad (1)$$

where $y(i)$ is the plant output; $u(i)$ is the control signal; $f(i)$ is the external disturbance ($|f(i)|\le f^*$, where f^* is specified); plant coefficients d_j and k_j ($j=\overline{0,n-1}$) are unknown values; n is known plant order; i is a sampling interval number during the total process (unless otherwise noted):

$$i=0,1,2,\ldots$$

Both analog-to-digital converter (ADC) reading the signal $y(i)$ and digital-to-analog converter (DAC) generating the signal $u(i)$ have their reference voltages. Therefore, the following conditions for the plant output and input should be held during the adaptation process:

$$|y(i)|\le y\_\,,\quad |u(i)|\le u\_\,. \qquad (2)$$

The control signal is generated by the controller:

$$\begin{aligned} &g_{n-1}u(n+i-1)+\ldots+g_1u(i+1)+g_0u(i)= \\ &=r_{n-1}\big(v(n+i-1)-y(n+i-1)\big)+\ldots \\ &+r_1\big(v(i+1)-y(i+1)\big)+r_0\big(v(i)-y(i)\big), \end{aligned} \qquad (3)$$

where $v(i)$ is a test signal; $r=[r_0, \ldots, r_{n-1}]$ and $g=[g_0, \ldots, g_{n-1}]$ are the coefficients of a controller, which is accessible for reading/setting via any connector.

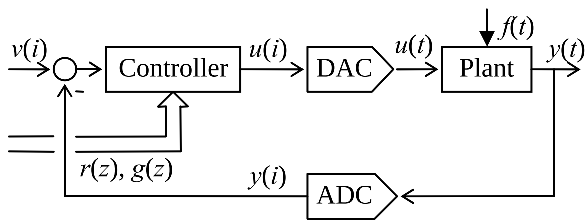

Figure 1. Block diagram of the closed-loop system

Thus, the source data for the ChAR-22 is:

a) y^* – declared control precision;
b) f^* – bound of the external disturbance;
c) $u\_$ and $y\_$ – bounds on both input and output signals;
d) n – plant order.

The problem is to adjust the parameters in (3) so that the precision requirement (i.e., purpose of the control) is satisfied:

$$|y(i)| \le y^*, \tag{4}$$

where y^* is specified.

3. Block diagram of ChAR-22

The ChAR-22 is adapted to use on the IBM-compatible computer (PC) which allows the use of both ADC and DAC converters. The ChAR-22, as well as the ChAR-21d, may be used also on another platform, such as DSP-processors with advanced architectures.

Figure 2 shows the block diagram of the system with the applied ChAR-22. The plant has the following discrete transfer function which consists of the plant coefficients (1):

$$W(z) = \frac{k_{n-1}z^{n-1} + \ldots + k_0}{z^n + d_{n-1}z^{n-1} + \ldots + d_0}. \tag{5}$$

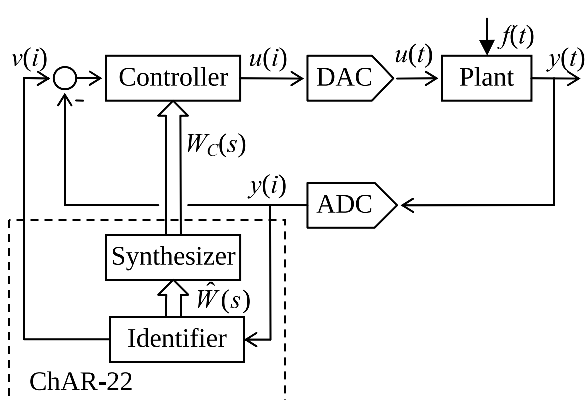

Figure 2. Block diagram of the system with applied ChAR-22

The ChAR-22 consists of two blocks: an identifier and a synthesizer. The identifier performs an active identification of the plant. Due to the identifier the following estimation of the transfer function (5) is determined:

$$\hat{W}(z) = \frac{\hat{k}_{n-1}z^{n-1} + \ldots + \hat{k}_0}{z^n + \hat{d}_{n-1}z^{n-1} + \ldots + \hat{d}_0}, \tag{6}$$

where $\hat{k}_j$ and $\hat{d}_j$ ($j = \overline{0, n-1}$) are estimates of the plant coefficients. The estimation (6) is a source data for the synthesizer. The synthesizer determines the following discrete transfer function of the controller based on its coefficients in (3):

$$W_C(z) = \frac{r_{n-1}z^{n-1} + \ldots + r_0}{g_{n-1}z^{n-1} + \ldots + g_0}. \tag{7}$$

After, the synthesizer tunes the controller with the given transfer function (7).

3.1 Identifier

Identifier applies the test signal $v(t)$ to the closed-loop system and reads the plant output $y(t)$ (via the ADC).

The test signal has the form of the poly-harmonic signal:

$$u(i) = \sum_{k=1}^{n} \rho_k \sin \omega_k ih, \quad i = \overline{0, lN-1}, \tag{8}$$

where $\rho = [\rho_{1,\ \ldots,}\ \rho_n]$ is the vector of test amplitudes; $\omega = [\omega_{1,\ \ldots,}\ \omega_n]$ is the vector of test frequencies; N is the number of samples in a test signal duration which is determined as:

$$N = \frac{\tau_b}{h},$$

where τ is the testing signal duration which is determined as:

$$\tau_b = \frac{2\pi}{\omega_l},$$

where ω_l is a lower bound of the test frequencies; a sample interval $h = 0.001$ by default, a number l is selected by a condition described below.

The lower bound ω_l is determined in process of the "self-tuning of the lower frequency" [9].

The test frequency vector ω is evaluated by means of the following formula:

$$\log \omega_k = \log \omega_l + (k-1)\frac{\log \omega_u - \log \omega_l}{n-1}, \quad k = \overline{1, n},$$

where an upper bound ω_u is evaluated by the following formula:

$$\omega_u = \frac{2\pi}{20h} = \frac{6.28319\text{rad.}}{0.01\text{sec.}} \approx 314 \text{ rad./sec.}$$

The test amplitudes vector ρ is self-tuned by a condition that both conditions (2) are held for both plant input $u(t)$ and plant output $y(t)$.

The plant output is applied to Discrete Fourier Transformation block (DFT) of the identifier. The DFT estimates the frequency parameters of the plant output:

$$\begin{cases} \widehat{\alpha}_k^{cl}(l) = \dfrac{2}{\rho_k lN} \displaystyle\sum_{i=0}^{N-1} y(i) \sin \omega_k ih \\ \widehat{\beta}_k^{cl}(l) = \dfrac{2}{\rho_k lN} \displaystyle\sum_{i=0}^{N-1} y(i) \sin \omega_k ih \end{cases} \quad k = \overline{1, n}. \tag{9}$$

First, the number $l = 1$. After, the estimations of the frequency parameters of the plant are computed by the following formulas:

$$\begin{cases} \widehat{\alpha}_k = \mathrm{Re}\left[\dfrac{\widehat{\alpha}_k^{cl} + j\widehat{\beta}_k^{cl}}{(1 - \alpha_k^{cl} - j\beta_k^{cl}) W_c(e^{j\omega h})} \right] \\ \widehat{\beta}_k = \mathrm{Im}\left[\dfrac{\widehat{\alpha}_k^{cl} + j\widehat{\beta}_k^{cl}}{(1 - \widehat{\alpha}_k^{cl} - j\widehat{\beta}_k^{cl}) W_c(e^{j\omega h})} \right] \end{cases} \quad k = \overline{1, n}, \tag{10}$$

where $W_C(s)$ is already known or given with known coefficients (3). Here j is imaginary unit.

Subsequently, the following $2n$ frequency equations set is solved:

$$\hat{k}(j\omega_k) - \hat{d}(j\omega_k)(\hat{\alpha}_k + j\hat{\beta}_k), \; k = \overline{1, n}. \tag{11}$$

As shown, the source data for the solving are the frequency parameters and the test frequencies. A result of the solving process is the estimated plant coefficients (6). Next, number l is being incremented; the test signal generator is applying its test signal to the system; the estimations (9) and, subsequently, (10) are being determined, and after, the estimations (6) are being found by the solution (11). The following value is computed:

$$\delta_g(l) = \max \left\{ \begin{matrix} \left| \dfrac{\alpha_k^{[l]} - \alpha_k^{[l-1]}}{\alpha_k^{[l]}} \right|, \left| \dfrac{\beta_k^{[l]} - \beta_k^{[l-1]}}{\beta_k^{[l]}} \right| \\ \left| \dfrac{k_{k-1}^{[l]} - k_{k-1}^{[l-1]}}{d_0^{[l]}} \right|, \left| \dfrac{d_{k-1}^{[l]} - d_{k-1}^{[l-1]}}{d_0^{[l]}} \right| \end{matrix} \right\}, \; k = \overline{1, n}.$$

Finally, the following condition is tested:

$$\delta_g(l) \le \varepsilon_g,$$

where ε_g is an accuracy of the plant coefficients (6) estimation, which is specified as:

$$\varepsilon_g = 0.05 \; (5\%).$$

If this condition hasn't been held, the number l is incremented and the test is repeated until a desired number δ_g^* will be found so that this condition holds.

3.2 Synthesizer

The synthesizer computes the controller coefficients so that the requirement (4) should be held. A synthesis algorithm is based on LQ-optimization with the following functional:

$$J = \sum_{i=0}^{N_{end}-1} \left\{ x^T(i) Q x(i) + u^2(i) + \varepsilon_1^2 \left[\frac{u(i+1) - u(i)}{h} \right]^2 + \ldots \right.$$
$$\left. + \varepsilon_\phi^2 \left[\frac{u(\phi + i) - u(\phi + i - 1)}{h} \right]^2 \right\},$$

where N_{end} is the number of samples in a considered duration; $x(i)$ is vector of the state variables of the identified plant (6); $\varepsilon_i \; (i = \overline{1, \psi})$ is a vector of relatively small coefficients; Q is the following positive defined matrix:

$$Q = qC^T C,$$

where q is given by the following formula:

$$q^2 = \frac{f^{*2}}{y^{*2}}.$$

Here C is a row vector of the identified plant (6) written in the state space form:

$$y = Cx + Du.$$

4. Experimental results

This section describes the experimental results of ChAR-22 algorithm. Along with the adaptation with the closed-loop system identification, the open-loop plant (without the feedback) identification was also applied in order to show an efficiency of the closed-loop adaptation. Therefore both controller and feedback are realized as the part of general code. The feedback might be open by the switch K (see Figure 3).

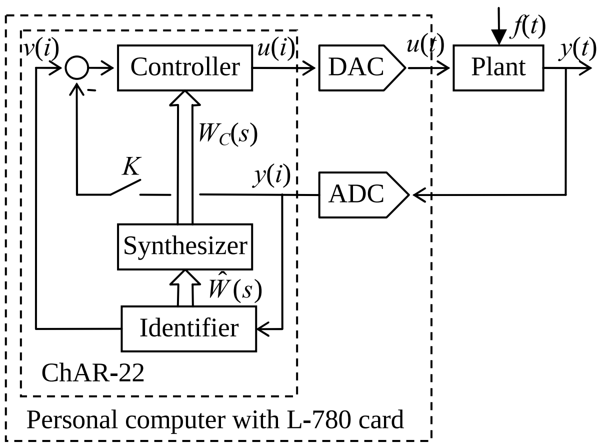

Figure 3. Block diagram of the used experimental setup

The experiment consisted of two stages. At the first stage of the experiment, the open-loop system was used (switch K was open and the transfer function of the controller was tuned as W_C = 1). At the second stage, since the discrete transfer function of the controller had been found, the controller was set by this transfer function. Also, at this stage, switch K was closed.

4.1 Experimental equipment and conditions

As shown in Figure 3, the setup consists of the IBM-compatible computer, the expansion card named L-780 (which contains both ADC and DAC) and the physical plant model (PPM). Both electrical schemes of the setup and the PPM are referenced in [9]. Figure 4 illustrates the used experimental setup.

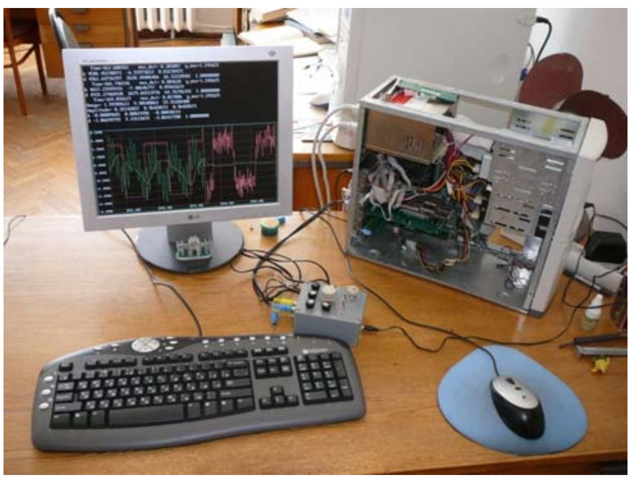
Figure 4. Experimental setup

The PPM is an electronic device which consists of some operational amplifiers so that it is described by 3-order differential equation. A source of the external disturbances is embedded within the PPM. A schematic circuit of the PPM is given in reference [9]. The PPM has the following continuous-time transfer function:

$$W(s) = \frac{4434.3}{s^3 + 63.095s^2 + 2521.4s + 4761.9} = \frac{0.9312}{0.00021s^3 + 0.01325s^2 + 0.5295s + 1}. \tag{12}$$

The following values were used for the source data:

a) $y^* = 0.05$;
b) $f^* = 1.7$ Volts;
c) $u\_ = y\_ = 5$ Volts;
d) $n = 3$.

Figure 5 shows the external disturbance $f(i)$ and the “native” plant output $\bar{y}(i)$ (when no test signal is applied to the system) during the experiment. We observed the following value for the open-loop plant output:

$$y_{\max}^{ol} \approx 0.92 \text{ Volts.}$$

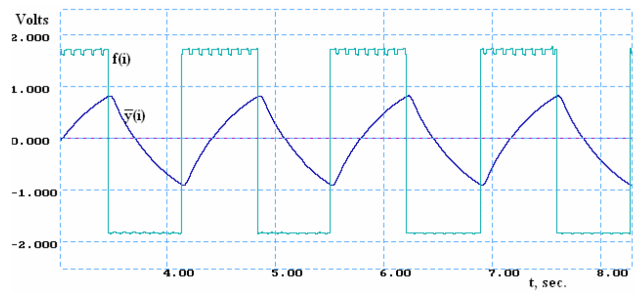

Figure 5. External disturbance and “native” plant output

4.2 Adaptation results for the open-loop plant

The typical plant output during the adaptation process is shown in Figure 6. One can see the following maximum for the plant output:

$$\left| y^{ol}(i) \right| \le 1.66 \text{ Volts,}$$

i.e., $y_{\max}$ is greater than $y_{\max}$ approximately by a factor of two.

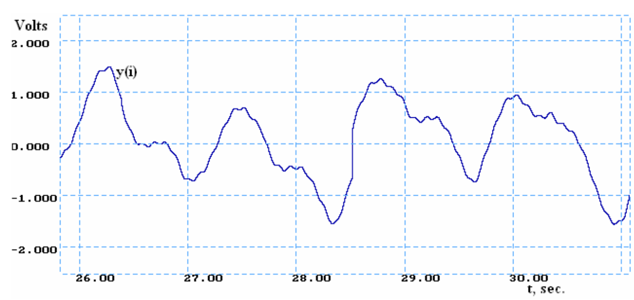

Figure 6. Plant output during the adaptation process

The adaptation process has given the following results. The controller coefficients (3) were synthesized during approximately 190 sec. (the first part of this time is the self-tunings of the test signal – 115 sec., the other part is the identification time – 75 sec.). The discrete transfer function corresponding to the following continuous was found:

$$\hat{W}(s) = \frac{15521}{s^3 + 156.5s^2 + 8662s + 16372} = \\ = \frac{0.00013s^2 - 0.009s + 0.948}{0.000061s^3 + 0.00955s^2 + 0.5284s + 1}. \tag{13}$$

Here a maximal error identification error equals 1.7% (compared to the required ε_g = 5%).

The result discrete transfer function of the controller is:

$$(13)\ \hat{W}_C(z) = \frac{-0.0023508z^2 + 0.0048436z - 0.00251525}{-1.174468z^3 + 3.33908z^2 - 3.1646z + 1}.$$

The output of the closed-loop system (closed by this controller) is shown in Figures 7 and 8. Comparing figures 5 and 8, one can see that error of the plant has been decreased approximately by a factor of eleven:

$$\left| y^{cl}(i) \right| \leq 0.08 \text{ Volts.}$$

Also, the identification time has been reduced more than in 6 times. Thus, the closing of the plant by this controller shows desired efficiency.

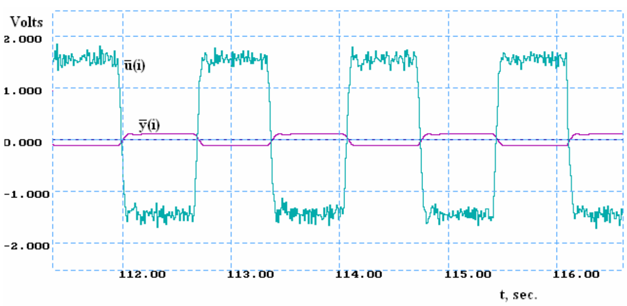

Figure 7. Plant input and output of the closed-loop system under the external disturbance

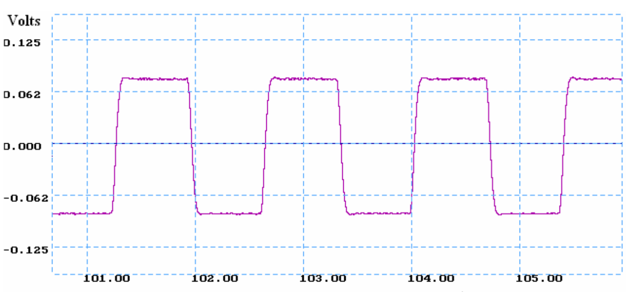

Figure 8. Disturbed plant output (zoomed in)

4.3 Result of the plant identification which is within the closed-loop system

The adaptation process took approximately 198 seconds (it is the pure identification time without any self-tunings). As a result, the following plant coefficients have been identified:

$$\hat{W}^{cl}(s) = \frac{1.12015s^2 - 92.4691s + 2329.22}{s^3 + 35.1966s^2 + 1455.14s + 2346.81} = \\ = \frac{0.00048s^2 - 0.0394s + 0.9925}{0.00043s^3 + 0.014997s^2 + 0.62s + 1}. \tag{14}$$

Here a maximal error identification error equals 9.1% (instead of the declared ε_g = 5%).

5. Conclusion

The ChAR-22 is written using the new finite frequency identification algorithm, and LG-optimization method. The used finite-frequency identification algorithm can be found in [10].

Using the ChAR-22 we could not achieve the declared level of the tolerance (4) ($y_{\max} = 0.08$ was achieved instead of the declared y^* = 0.05). The same result was obtained for the experiment described in [9] ($y_{\max} = 0.0045$ was achieved instead of the declared y^* = 0.0025). Hence this fact is not concerned with identification algorithm. Thus the efficiency of the using of the finite-frequency identification algorithm, proposed in [10], is evident.

The paper shows that the efficiency of the adaptation depends of the feedback presence. After the closing of the plant, the identification time increased in 2.5 times. Furthermore, the identification error was in two times greater than declared. This fact could cause the significant error of the synthesis and, hence, the reduction of the efficiency. Thus the using of the given finite-frequency algorithm applying to the closed-loop system may cause the reduction of the frequency adaptive controller efficiency.

References

[1] B.D.O. Anderson, R.R. Bitmead, C.R. Jonson, P.V. Kokotovich, R. Kosut, I.M.J. Mareles, L. Rralu and B.D. Riedle, Stability of adaptive system passivity and abereging analysis. *MIT Press*, Cambridge, Massachusets, 1986.

[2] R. Iserman, Digital control systems. Springer Verlag, (1981).

[3] I.D. Landau, Competition, interaction, control. *European Control Conference, Plenaries and mini-courses*, 1997, 1-35.

[4] V.A. Yakubovich, Adaptive stabilization of continuous linear plants. *Automation and Remote Control,* Vol. 49, No 4, 1981, 97-107.
[5] X. Zhao, X. and R. Lozano, Adaptive pole placement for continuous-time system in the presence of bounded disturbance. *12th World Congress IFAC. Preprints of papers,* Vol. 1, 1993, 205-210.
[6] A.G. Alexandrov, Frequency adaptive control of stable plant in the presence of bounded disturbance. *IFAC Workshop Adaptive Systems in Control and Signal Processing. Preprints,* 1998, 94-99.
[7] A.G. Alexandrov, Ju. F. Orlov, Frequency adaptive control of multivariable plants. *15th world congress of IFAC, Preprints,* Barcelona, 2002.
[8] A.G. Alexandrov, Finite-frequency identification: selftuning of test signal. *16th world congress of IFAC, Preprints,* Praha, 2005.
[9] A.G. Alexandrov, D.G. Karikov, E.Y. Kuritsyna, Frequency Adaptive Controller CHAR-21. *Proceedings of international conference "System Identification and Control Problems",* Moscow, 2006, 2361-2381.
[10] A. Alexandrov, D. Khomutov, Ks. Shchegoleva, Dynamic system identifier "DPI-3". Experimental investigations. *Proceedings of the 27th IASTED international conference on "Modelling, Identification and Control" (MIC 2008),* Innsbruck, Austria, 2008.