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Self-Tuning PID-I Controller

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Abstract: The plant of the first order with time delay is considered. Coefficients of plant are unknown and can be changed in some isolated time moments. Self-tuning PID-I controller for the plant is proposed in the presence of unknown-but-bounded external disturbances. The test signal with sum of two harmonics is used for identification of plant coefficients. Methods of tuning of amplitudes and frequencies are given. To guarantee the stability of the closed-loop system the special switching technique between I- and PID-controllers is presented. The I-controller is selected to provide stability of the plant independently on its mode. It provides stability and allows to identify the plant. Algorithms of self-tuning of PID- and I-controllers are proposed. A real regulator named *ST-PID-1* was developed on the basis of these algorithms. The obtained results are supported by experimental applications.

Keywords: adaptive control, PID control, I-controller, time delay, frequency identification, unknown bounded disturbance.

1. INTRODUCTION

Proportional-Integral-Differential (PID) controllers are widely used in industry because they are simple and effective. Self-tuning PID controllers are designed for plants with time-varying coefficients. Different methods of the identification are used for plant identification. These methods allow to estimate plant's parameters and use them for adjusting of PID controller. However, the identification is complicated because of external disturbances. Step response methods (Ziegler and Nichols (1942)) are used when the external disturbance is absent. In this case, it is also possible to use relay methods (Ziegler and Nichols (1942), Astrom and Hagglund (1984), C.C. Hang (1993), Astrom and Hagglund (2006)). However, the relay method is not suitable because it breaks the normal mode of the plant. In papers Sato and Inoue (2005) and Sato and Kameoka (2008), the least-squares method (LSM) is used for determination of coefficients of the plant. In the paper W.K. Ho (1996), the same method is used in conjunction with band-pass filter. This method allows better detection of coefficients of the plant. However, LSM is not applicable when the external disturbance is an unknown bounded function. It is easy to find an external disturbance for LSM which gives impermissible identification errors.

In this paper, the self-tuning PID-I controller is proposed. Coefficients of plant are unknown and they change in sufficiently seldom time moments. The identification is complicated due to external disturbances. The finite-frequency identification method is used for identification (Alexandrov (1994), Alexandrov (1999)). The plant is excited by a test signal which is a sum of two harmonics. Amplitudes of the test signal is adjusted so that the level of distortions introduced in the plant's output does not exceed the specified limit. Fourier's filter is used. Identification error for given filtration time depends on the choice of test signal frequencies (see, for example, Alexandrov (2005)). The frequencies must be chosen in such way that minimizes an identification error of the given filtration time.

The results of identification are used for design of PID controller based on concept of Internal Model Principle (Visioli (2002)). This controller compensates the time constant of the plant, so the system performance is determined by the given parameter of synthesis of controller and time delay in the control channel. PID controller also provides high amplitude and phase margins.

The closed loop system with PID controller may lose stability because of change of plant coefficients. In this case, the plant is closed by I-controller instead of PID controller. I-controller can provide stability for a large range of plant's coefficients and it also allows tracking a reference signal without static error. On the other hand, this controller can not provide fast reference tracking performance. So the plant is closed by I-controller only when the closed loop with PID controller loses stability. Thus, the loop of the system is not broken. It allows to identify the plant.

The paper is organized as follows. In the next section the problem statement and the basic assumptions are presented. The Section 3 is devoted to an identification problem of the plant by the finite-frequency method. Relation of identification errors with test signal frequencies is investigated. Methods of self-tuning of amplitudes, frequencies and duration of filtration are proposed. Then an expression for coefficient of I-controller is given and algorithms of self-tuning of PID-I controller are proposed. In Section 5, real self-tuning controller named *ST-PID-1*, which implement on the basis of these algorithms, is described. Experimental investigations of controller are given.

2. PROBLEM STATEMENT

Consider a plant described by equation

$$T^{[i]}\dot{y}(t) + y(t) = k_p^{[i]} u(t - \tau^{[i]}) + f(t), \qquad (1)$$
$$t^{[i]} \le t < t^{[i+1]}, \quad i = 1, 2, ..., N,$$

where $y(t)$ and $u(t)$ are output and input of the plant respectively, $f(t)$ is an unknown-but-bounded external disturbance ($|f(t)| \le f^*$), i is the number of the plant's mode ($i = 1, 2, ...N$). Coefficients $k_p^{[i]}$, $T^{[i]}$, $\tau^{[i]}$ are unknown numbers, they change in known (for simplicity) time moments $t^{[1]}$, $t^{[2]}$, ..., $t^{[N]}$, and they are constant in each i-th mode

$$t^{[i]} \le t < t^{[i+1]}, \quad i = 1, 2, ..., N. \qquad (2)$$

Length of intervals (2) is such that $t^{[i+1]} - t^{[i]} > \Delta t^*$ ($i = 1, 2, ..., N$) where Δt^* is the sufficiently large positive number.

The possible values of plant's coefficients lie into intervals

$$\underline{k_p} \le k_p^{[i]} \le \overline{k_p}, \;\; \underline{T} \le T^{[i]} \le \overline{T}, \;\; \underline{\tau} \le \tau^{[i]} \le \overline{\tau}, \qquad (3)$$
$$i = 1, 2, ..., N,$$

where lower ($\underline{k_p}, \underline{T}, \underline{\tau}$) and upper ($\overline{k_p}, \overline{T}, \overline{\tau}$) bounds are given positive numbers.

The PID controller is

$$g^{[i]}\dot{u}(t) + u(t) = k_c^{[i]} \varepsilon^{[i]}(t) + k_i^{[i]} \int_0^t \varepsilon^{[i]}(\tilde{t}) d\tilde{t} + k_d^{[i]} \dot{\varepsilon}^{[i]}(t),$$
$$t_{st}^{[i]} \le t < t_{st}^{[i+1]}, \;\; t^{[i]} \le t_{st}^{[i]} < t^{[i+1]}, \;\; i = 1, 2, ..., N. \qquad (4)$$

$$\varepsilon^{[i]}(t) = y_{sp}(t) - y(t) + v^{[i]}(t), \qquad (5)$$

where $g^{[i]}$, $k_c^{[i]}$, $k_i[i]$, $k_d^{[i]}$ are coefficients of the PID controller, they are changing in the time moments $t_{st}^{[i]}$, $\varepsilon(t)$ is the tracking error, $y_{sp}(t)$ is the reference signal, $v(t)$ is the test signal.

Modes of plant and PID controller are illustrated in picture 1.

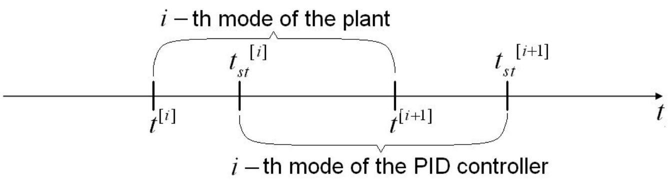

Fig. 1. Time intervals of plant and PID controller

Note, there does not exist a PID controller with time invariant coefficients that could provide stability for plant in each mode.

Tracking error (5) must satisfy the following condition:

$$|\varepsilon^{[i]}(t)| = |\varepsilon^{[i]*}(t)| + |\xi^{[i]}(t)|, \;\; t \ge t_{st}^{[i]}, \;\; i = 1, 2, ..., N, \;\; (6)$$

where $|\varepsilon^{[i]*}(t)|$ is the achieved tracking error (ideal tracking error) in case when the plant in i-th mode is known. Values $|\xi^{[i]}(t)|$ must satisfy the condition:

$$|\xi^{[i]}(t)| < q|\varepsilon^{[i]*}(t)|, \;\; i = 1, 2, ..., N, \qquad (7)$$

where q is the sufficiently small positive number.

Coefficients of the PID controller (4) are calculated through the following expressions (Visioli (2002))

$$k_c^{[i]} = \frac{2T^{[i]} + \tau^{[i]}}{2k_p^{[i]}(\lambda^{[i]} + \tau^{[i]})}, \;\; k_i^{[i]} = \frac{1}{k_p^{[i]}(\lambda^{[i]} + \tau^{[i]})},$$
$$k_d^{[i]} = \frac{T^{[i]}\tau^{[i]}}{2k_p^{[i]}(\lambda^{[i]} + \tau^{[i]})}, \;\; g^{[i]} = \frac{\lambda^{[i]}\tau^{[i]}}{2(\lambda^{[i]} + \tau^{[i]})}, \qquad (8)$$
$$i = 1, 2, ..., N,$$

where $\lambda^{[i]}$ is the design parameter. It is chosen as $\lambda^{[i]} = \frac{T^{[i]}}{\o}$, where $\o = 2 \div 4$ (Visioli (2002), Astrom and Hagglund (2006)).

The closed loop system (1), (4), (5) is approximately described by the following differential equation

$$\lambda^{[i]}\dot{y}(t) + y(t) = y_{sp}(t - \tau^{[i]}). \qquad (9)$$

Influence of the test signal $v(t)$ to the output of the plant is bounded and described by the influence coefficient of test signal:

$$K_v^{[i]} = \sqrt{\frac{\int_{t_0^{[i]}+t_a}^{t_0^{[i]}+2t_a} (y_{sp}(t) - y(t))^2 \, dt}{\int_{t_0^{[i]}}^{t_0^{[i]}+t_a} (y_{sp}(t) - \overline{y}(t))^2 \, dt}}, \;\; i = 1, 2, ..., N, \;\; (10)$$

where $y(t)$ is the output of the closed loop system (1), (4), (5), $\overline{y}(t)$ is the output of the closed loop system (1), (4), (5) when the test signal is absent ($v(t) = 0$), t_a is the sufficiently large time.

The influence coefficient of test signal $K_v^{[i]}$ satisfies the following condition

$$K_v^{[i]} \le K_v^*, \;\; i = 1, 2, ..., N, \qquad (11)$$

where K_v^* is the specified tolerance on the effect of test signal.

The problem is to find coefficients of PID controller in each i-th mode such that the conditions (6) and (11) are satisfied.

3. IDENTIFICATION OF THE PLANT

The problem of finding of PID controller coefficients is reduced to identification coefficients of plant (1) in each i-th mode. PID parameters are calculated (8) through of plant coefficients. So it is important to identify the coefficients of the plant more accurately because the effectiveness of self-tuning of PID-controller depends on the accuracy of the identification results.

3.1 Finite-frequency identification

There are some difficulties in the identification of plant:

a) reference signal is often a constant function therefore the input signal has not enough harmonics (Ljung (1987));

b) often, the external disturbance is an unknown-but-bounded function;

c) the plant must be identified in the closed loop (1), (4), (5).

The above problems cab be solved by using the finite-frequency identification method. In accordance with this method numbers

$$\alpha_k^{[i]} = \text{Re } W_p^{[i]}(j\omega_k), \;\; \beta_k^{[i]} = \text{Im } W_p^{[i]}(j\omega_k), \;\; k = 1, 2, \tag{12}$$

where

$$W_p^{[i]}(s) = \frac{k_p^{[i]} e^{-\tau^{[i]} s}}{T^{[i]} s + 1}, \;\; i = 1, 2, ..., N, \tag{13}$$

are called *frequency domain parameters* (FDP) (Alexandrov (1994)).

The FDP estimates are determined experimentally as follows: after the closed loop system is excited by the test signal

$$v^{[i]}(t) = \rho_1^{[i]} \sin \omega_1 t + \rho_2^{[i]} \sin \omega_2 t, \tag{14}$$

where $\rho_k^{[i]}$ and test frequencies ω_k $(k = 1, 2)$ are specified positive numbers, test frequencies are multiples of each other $\omega_2 = \mu\omega_1$ $(1 < \mu < \infty$, μ is integer), plant's input $u(t)$ and output $y(t)$ are fed to the Fourier filters, whose outputs give the following estimates

$$\begin{aligned}
\hat{\alpha}_{yk}^{[i]} &= \alpha_{yk}^{[i]}(\bar{t}^{[i]}) = \tfrac{2}{\rho_k^{[i]}\bar{t}^{[i]}} \int_{t_F^{[i]}}^{t_F^{[i]}+\bar{t}^{[i]}} y(t) \sin \omega_k t dt, \\
\hat{\beta}_{yk}^{[i]} &= \beta_{yk}^{[i]}(\bar{t}^{[i]}) = \tfrac{2}{\rho_k^{[i]}\bar{t}^{[i]}} \int_{t_F^{[i]}}^{t_F^{[i]}+\bar{t}^{[i]}} y(t) \cos \omega_k t dt, \\
\hat{\alpha}_{uk}^{[i]} &= \alpha_{uk}^{[i]}(\bar{t}^{[i]}) = \tfrac{2}{\rho_k^{[i]}\bar{t}^{[i]}} \int_{t_F^{[i]}}^{t_F^{[i]}+\bar{t}^{[i]}} u(t) \sin \omega_k t dt, \\
\hat{\beta}_{uk}^{[i]} &= \beta_{uk}^{[i]}(\bar{t}^{[i]}) = \tfrac{2}{\rho_k^{[i]}\bar{t}^{[i]}} \int_{t_F^{[i]}}^{t_F^{[i]}+\bar{t}^{[i]}} u(t) \cos \omega_k t dt, \\
&k = 1, 2, \;\; i = 1, 2, ..., N,
\end{aligned} \tag{15}$$

where $\bar{t}^{[i]}$ is a filtering time and $t_F^{[i]}$ is the initial instant for filtering. Numbers $\bar{t}^{[i]}$ and $t_F^{[i]}$ are multiples of a base period $T_b = \frac{2\pi}{\omega_1}$ and satisfied the inequality $t^{[i]} < t_F^{[i]} + t^{[i]} < t^{[i+1]}$. The numbers $\hat{\alpha}_{yk}^{[i]}$, $\hat{\beta}_{yk}^{[i]}$, $\hat{\alpha}_{uk}^{[i]}$, $\hat{\beta}_{uk}^{[i]}$ $(k = 1, 2)$ allow us to estimate the plant model coefficients.

If the disturbance $f(t)$ and reference signal $y_{sp}(t)$ are strongly FF-filterability, this mean that disturbance $f(t)$ and reference signal $y_{sp}(t)$ does not contain test frequencies ω_1, ω_2, then

$$\begin{aligned}
&\lim_{\bar{t}^{[i]}\to\infty} \alpha_{yk}^{[i]}(\bar{t}^{[i]}) = \alpha_{yk}^{[i]}, \;\; \lim_{\bar{t}^{[i]}\to\infty} \beta_{yk}^{[i]}(\bar{t}^{[i]}) = \beta_{yk}^{[i]}, \\
&\lim_{\bar{t}^{[i]}\to\infty} \alpha_{uk}^{[i]}(\bar{t}^{[i]}) = \alpha_{uk}^{[i]}, \;\; \lim_{\bar{t}^{[i]}\to\infty} \beta_{uk}^{[i]}(\bar{t}^{[i]}) = \beta_{uk}^{[i]}, \\
&k = 1, 2, \;\; i = 1, 2, ..., N,
\end{aligned}$$

where $\alpha_{yk}^{[i]}$, $\beta_{yk}^{[i]}$, $\alpha_{uk}^{[i]}$, $\beta_{uk}^{[i]}$ $(k = 1, 2)$ FDP of the closed loop system (Alexandrov (1998)):

$$\begin{aligned}
\alpha_{yk}^{[i]} + j\beta_{yk}^{[i]} &= \tfrac{W_c^{[i]}(j\omega_k) W_p^{[i]}(j\omega_k)}{1+W_c^{[i]}(j\omega_k) W_p^{[i]}(j\omega_k)}, \\
\alpha_{uk}^{[i]} + j\beta_{uk}^{[i]} &= \tfrac{W_c^{[i]}(j\omega_k)}{1+W_c^{[i]}(j\omega_k) W_p^{[i]}(j\omega_k)}, \\
&k = 1, 2, \;\; i = 1, 2, ..., N,
\end{aligned} \tag{16}$$

where $W_c^{[i]}(j\omega_k)$ is the frequency transfer function of the PID controller. Conditions of FF-filterability can be examined by experiment by using the Fourier filter (15) without test signal ($v^{[i]}(t) = 0$). Condition of strongly FF-filterability is satisfied when outputs of filter are zero (See Alexandrov (2005)).

Assertion 3.1. Numbers $\alpha_{yk}^{[i]}$, $\beta_{yk}^{[i]}$, $\alpha_{uk}^{[i]}$, $\beta_{uk}^{[i]}$ $(k = 1, 2)$ *are related with FDP* $\alpha_k^{[i]}$, $\beta_k^{[i]}$ $(k = 1, 2)$ *as follows*

$$\alpha_k^{[i]} = \frac{\alpha_{yk}^{[i]}\alpha_{uk}^{[i]} + \beta_{yk}^{[i]}\beta_{uk}^{[i]}}{(\alpha_{uk}^{[i]})^2 + (\beta_{uk}^{[i]})^2}, \;\; \beta_k^{[i]} = \frac{-\alpha_{yk}^{[i]}\beta_{uk}^{[i]} + \beta_{yk}^{[i]}\alpha_{uk}^{[i]}}{(\alpha_{uk}^{[i]})^2 + (\beta_{uk}^{[i]})^2}, \tag{17}$$

$$k = 1, 2, \;\; i = 1, 2, ..., N.$$

■

Assertion 3.2. Coefficients of the plant (1) and FDP (12) are related by following expressions (for simplicity, the index $[i]$ *omitted)*

$$T^2 = \frac{(\alpha_2^2 + \beta_2^2) - (\alpha_1^2 + \beta_1^2)}{\omega_1^2(\alpha_1^2 + \beta_1^2) - \omega_2^2(\alpha_2^2 + \beta_2^2)}, \tag{18.a}$$

$$k_p^2 = (\alpha_2^2 + \beta_2^2)(T^2\omega_2^2 + 1), \tag{18.b}$$

$$\tau = \frac{1}{\omega_1} atan \frac{T\omega_1\alpha_1 + \beta_1}{T\omega_1\beta_1 - \alpha_1} \tag{18.c}$$

$$\omega_1\tau < \frac{\pi}{2} \tag{18.d}$$

■

3.2 Choice of frequencies of the test signal

Arbitrary choice of frequencies of the test signal can lead to large identification errors for given filtration time (Alexandrov (2005)). Let's analyze errors of identification.

Let FDP β_1 in equation (18.a) be determined with error δ_β:

$$\hat{\beta}_1 = \beta_1 + \delta_\beta. \tag{19}$$

Introduce a quantity that characterizes the error in determining of the time constant:

$$\Delta_{T^2} = \hat{T}^2 - T^2 = \frac{(\alpha_2^2 + \beta_2^2) - (\alpha_1^2 + (\beta_1 + \delta_\beta)^2)}{\omega_1^2(\alpha_1^2 + (\beta_1 + \delta_\beta)^2) - \omega_2^2(\alpha_2^2 + \beta_2^2)} - T^2. \tag{20}$$

Since $\omega_2 = \mu\omega_1$, $1 < \mu < \infty$, then the following assertions.

Assertion 3.3. If frequency of the test signal tends to zero $(\omega_1 \to 0)$ *then* $\Delta_{T^2} \to \infty$. ■

Assertion 3.4. There are exist number δ_β, T, τ, k_p, μ ω_1 *such that, quantity* Δ_{T^2} *is unacceptably large.* ■

Assertion 3.4 imposes the requirement of a sufficiently precise definition of the FDP. In this case the error δ_β is sufficiently small and there doesn't exist numbers T, τ, k_p, μ and ω_1 when quantity Δ_{T^2} is large.

An single-harmonic test signal uses for an first-order plant without time delay. In this case, the frequency must be chosen as close as possible to the magnitude of the inverse time constant $\frac{1}{T}$ (Alexandrov (2005)). With this in mind, as well as assuming that $\tau^{[i]} \leq T^{[i]}$, $(i = 1, 2, ..., N)$, we choose the frequencies as follows

$$\omega_1 = \frac{1}{2\overline{T}}, \;\; \omega_2 = 2\omega_1. \tag{21}$$

3.3 Self-tuning of amplitudes of the test signal

Amplitudes are self-tuned in the each i-th mode of plant by using similar Alexandrov (2005) algorithm. The amplitudes are computed as

$$\rho_k^{[i]} = \rho_b \omega_k, \quad k = 1, 2, \;\; i = 1, 2, ..., N, \tag{22}$$

where ρ_b is the base amplitude (first, it is chosen sufficiently small).

The purpose of self-tuning is a coefficient of influence of test signal should satisfy

$$K_v^* - \Delta \leq K_v^{[i]} \leq K_v^*, \;\; i = 1, 2, ..., N, \tag{23}$$

where Δ is the given sufficiently small number.

Amplitudes are self-tuned by the following algorithm. *Algorithm 3.1*

(1) Calculate amplitudes (22) with sufficiently small ρ_b and then feed to the closed loop system (1), (4), (5) signal (14) with given test frequencies (21).

(2) Examine the condition (23), if it is satisfied then turn off the test signal and stop the self-tuning of amplitudes. On the other hand, if the condition (23) is not satisfied then put $\rho_b = \rho_b \cdot \sigma$, where σ is the given positive number, and so on until the condition (23) will be satisfied.

3.4 Duration of the identification

The identification in each i-th mode stops when the following conditions are satisfied (for simplicity, index $[i]$ omitted)

$$\left| \frac{\hat{T}^{\kappa T_b} - \hat{T}^{(\kappa-1)T_b}}{\hat{T}^{\kappa T_b}} \right| \leq \theta, \; \left| \frac{\hat{k_p}^{\kappa T_b} - \hat{k_p}^{(\kappa-1)T_b}}{\hat{k_p}^{\kappa T_b}} \right| \leq \theta, \tag{24}$$

$$\left| \frac{\hat{\tau}^{\kappa T_b} - \hat{\tau}^{(\kappa-1)T_b}}{\hat{\tau}^{\kappa T_b}} \right| \leq \theta,$$

$\hat{T}^{\kappa T_b}$, $\hat{k_p}^{\kappa T_b}$, $\hat{\tau}^{\kappa T_b}$, $\kappa = 1, 2, ..., M$ estimates are given at time moments multiples of the base period T_b, θ is a given positive number. Initial conditions are $\hat{T}^0 = 0$, $\hat{k_p}^0 = 0$, $\hat{\tau}^0 = 0$. It is assumed that the conditions (24) are satisfied before will come next $(i+1)$-th mode.

4. I-CONTROLLER AND SELF-TUNING ALGORITHM OF PID/I CONTROLLER

4.1 I-controller

The plant in $(i+1)$-th mode closed by PID-controller, which designed for the plant of i-th mode, may lose stability. In this case, for identification, the plant is closed by I-controller. I-controller (it follows from (4) with $g = k_c = k_d = 0$) is

$$u(t) = k_i \int_{t_0}^{t} \varepsilon(t) dt \tag{25}$$

where k_i is constant for all modes.

Assertion 4.1. The closed loop system (1), (25), (5) is stable if

$$0 < k_i < \frac{l_m}{\overline{k_p}}, \tag{26}$$

where

$$l_m = \min_{\substack{\underline{\tau} \leq \tau \leq \overline{\tau} \\ \underline{T} \leq T \leq \overline{T}}} \frac{\omega_u}{\sin \omega_u \tau} \tag{27}$$

under conditions

$$T \omega_u \sin \omega_u \tau = \cos \omega_u \tau, \tag{28}$$

$$0 < \omega_u < \frac{\pi}{2\tau}. \tag{29}$$

■

4.2 Self-tuning algorithm of PID/I controller

1) Close the plant by I-controller (25) in the first mode. Frequencies of test signal are calculated by using (21);

2) Find amplitudes of test signal are self-tuned by using *Algorithm 3.1*;

3) Identify the plant in the closed loop system with I- (or PID-controller): a) turn on the test signal (14) and feed input and output of the plant to the Fourier's filter (15) which output for a given value $\bar{t}$ (or if (24) is satisfied) gives the estimates of FDP (17); b) Use (18.a)-(18.c), substituting estimates of FDP, for calculation of plant's coefficients estimates and then turn off the test signal ($v(t) = 0$);

4) Coefficients of PID-controller for identified plant are calculated by using (8). Then the plant is closed by this PID controller.

5) There are two variants into the next mode of plant: a) if condition $|y(t)| \leq y^*$ is satisfied (the closed loop system is stable) then go to operation 2); b) otherwise, if the closed loop system loses stability ($|y(t)| > y^*$) then go to operation 1).

5. EXPERIMENTAL RESULTS

5.1 Experimental setup FM-2

Experimental setup FM-2 is the setup for investigations of adaptive controllers in a semi-industrial environment. This setup includes an industrial controller WinCon-8341 and industrial computer Athena, which interact with each other through embedded DAC and ADC converters. Plant simulator is implemented in the industrial computer Athena. Self-tuning PID-I controller, called ST-PID-1, is implemented in the industrial controller WinCon-8341.

5.2 Results of experiments

Simulated plant have a follow form:

$$w_p(s) = \frac{k_p^{[i]} e^{-\tau^{[i]} s}}{(T^{[i]} s + 1)(T_1^* s + 1)(T_2^* s + 1)}, \;\; i = 1, 2, ..., N, \tag{30}$$

where T_1^* and T_2^* - unmodeled dynamics $T_1^* \leq T_2^* < T$. In experiments: $T_1^* = 0.2$ sec and $T_2^* = 0.3$ sec. Coefficients of the plant (30) $k_p^{[i]}$, $T^{[i]}$, $\tau^{[i]}$ changed in each i-th mode according with table 1. Duration of an each mode is 1400 second. External disturbance is $f(t) = 0.5 \text{sign}[\sin 3t]$. Bounds of coefficients are $\underline{k_p} = 0.1$, $\overline{k_p} = 4$, $\underline{T} = 1$, $\overline{T} = 8$, $\underline{\tau} = 0.1$, $\overline{\tau} = 2$.

Table 1. Coefficients of the plant

	1	2	3	4	5	6	7	8
k_p	3.51	2.73	2.16	1.05	1.49	3.97	3.89	2.29
T	3.22	1.49	2.20	6.90	2.63	3.69	2.53	6.13
τ	0.61	0.47	1.75	1.16	1.33	0.38	1.25	1.27

Experiment 1. PID controller designed for minimal bounds $(\underline{k}_p, \underline{T}, \underline{\tau})$. Parameters of PID controller doesn't change, self-tuning doesn't work. Results of experiment are shown in picture 2.

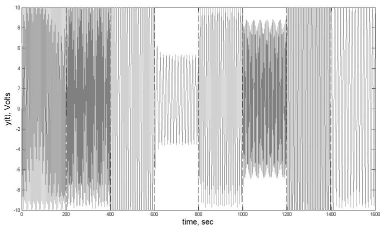

Fig. 2. Output of the system without selftuning

From picture 2 shown that PID controller designed for minimal bounds can't provide stability for all modes.

Experiment 2. Self-tuning PID/I controller is work. Test frequencies are $\omega_1 = 0.0625$rad/s and $\omega_2 = 0.1250$rad/s. Amplitudes self-tuning with the influence coefficient of test signal $K_v^* = 1.3$ and the tolerance $\Delta = 0.1$. PID controller designed by (8) with $\lambda^{[i]} = \frac{\hat{T}^{[i]}}{4}$ $(i = 1, 2, ..., 8)$. The identification stops when the relative identification error is $\theta = 0.02$.

Output of the system is showed in picture 3 a). The function $|\xi^{[i]}(t)|$ from (6) $(|\xi^{[i]}(t)| = |\varepsilon^{[i]}(t)| - |\varepsilon^{[i]*}(t)|)$ is shown at 3 b). I-controller is connecting in the 5-th mode, it shown in picture 3 c). The function $|\xi^{[i]}(t)|$ in 6-th mode is shown in 3 d). Notations are used in all figures: black vertical dash line denote time moments $t^{[i]}$, black small vertical dash dot line denote time moments $t_{st}^{[i]}$. Influence coefficient of test signal $K_v^{[i]}$ and estimates are shown in table 2, where gray rows are shows modes when I-controller connected.

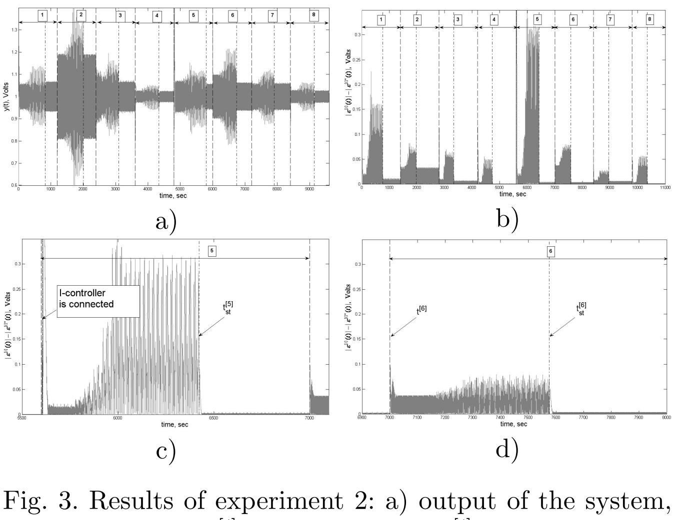

Fig. 3. Results of experiment 2: a) output of the system, b) function $|\xi^{[i]}(t)|$, c) function $|\xi^{[i]}(t)|$ in 5-h mode, d) function $|\xi^{[i]}(t)|$ in 6-h mode.

Table 2. Results of experiment 2

Mode number	k_p	$\hat{k_p}$	T, c	$\hat{T}$	τ, c	$\hat{\tau}$	$K_v^{[i]}$
1	3.51	2.90	3.22	2.17	0.61	1.81	1.23
2	2.73	2.70	1.49	0.21	0.47	0.25	1.23
3	2.16	1.67	2.20	1.86	1.75	1.31	1.16
4	1.05	0.89	6.90	5.88	1.16	2.07	1.20
5	1.49	1.42	2.63	2.13	1.33	2.27	1.29
6	3.97	3.39	3.69	3.40	0.38	0.36	1.11
7	3.89	4.14	2.53	2.73	1.25	1.28	1.13
8	2.29	1.60	6.13	5.57	1.27	1.63	1.16

We can draw conclusions: from figures 3 b), c) and d) that purpose (6) is satisfied; from last column of table 2 that purpose (11) is satisfied.

6. CONCLUSION

In this paper, a new technique of adaptive control of the multi-mode first order plant with time delay has been presented. It is based on two-frequencies identification of the plant and uses PID- and I-controllers. Formulas for calculation of plant's coefficients using plant's input and output are given. Relations of identification errors with frequencies of test signal are investigated and these frequencies are determined. The method of self-tuning of amplitudes of test signal with the specified influence coefficient of test signal is given. To provide the correct stabilizing process after the switching time moment instead of PID-controller the I-controller is implemented in the case on unstable behavior. On the developed method the so-called *ST-PID-1* controller is realized. The results of experimental investigations are demonstrated effectiveness of ST-PID-1.

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Appendix A. PROOF OF ASSERTION 3.1

Eliminating $W_c^{[i]}$ from (16) and noting that $W_p^{[i]}(j\omega_k) = \alpha_k^{[i]} + j\beta_k^{[i]}$, $(k = 1, 2)$, we obtain the following relations

$$\alpha_{yk}^{[i]} + j\beta_{yk}^{[i]} = (\alpha^{[i]} + j\beta^{[i]})(\alpha_{uk}^{[i]} + j\beta_{uk}^{[i]}),$$
$$k = 1, 2, \quad i = 1, 2, ..., N.$$

Solving these relations with respect to $\alpha_k^{[i]}$, $\beta_k^{[i]}$ $(k = 1, 2)$ we get relations (17).

Appendix B. PROOF OF ASSERTION 3.2

In fact, from the frequency transfer function of the plant

$$w_p(j\omega_k) = \frac{k_p(\cos\omega_k\tau - j\sin\omega_k\tau}{1 + jT\omega_k} = \alpha_k + j\beta_k, \; k = 1, 2, \tag{B.1}$$

follows

$$k_p\cos\omega_k\tau = -T\beta_k\omega_k + \alpha_k, \tag{B.2}$$
$$-k_p\sin\omega_k\tau = T\omega_k\alpha_k + \beta_k, \quad k = 1, 2.$$

Eliminating the time delay, we obtain

$$k_p^2 - (\alpha_k^2 + \beta_k^2)\omega_k^2 T^2 = \alpha_k^2 + \beta_k^2, \quad k = 1, 2.$$

Solution of above equations gives (18.a) and (18.b). In order to obtain the time delay, we divide second equation in (B.2) by the first equation, it gives

$$\tan\omega_k\tau = \frac{T\omega_k\alpha_k + \beta_k}{T\omega_k\beta_k - \alpha_k}, \quad k = 1, 2. \tag{B.3}$$

If we assume that the frequency of test signal is chosen so that condition (18.d) is satisfied, then the solution (B.3) for k = 1 gives (18.c).

Appendix C. PROOF OF ASSERTION 3.3

After transformation (20), we have

$$\Delta_T = -\frac{\nu(\frac{1}{\omega_1^2} + T^2)}{1 + \nu}, \tag{C.1}$$

where

$$\nu = \frac{2\beta_1\delta_\beta + \delta_\beta^2}{(\alpha_1^2 + \beta_1^2)\omega_1^2 - (\alpha_2^2 + \beta_2^2)\mu^2\omega_1^2}.$$

Expression (B.1) gives following equations

$$\beta_1 = -\frac{k_p(T\omega_1\cos\omega_1\tau + \sin\omega_1\tau)}{T^2\omega_1^2 + 1},$$
$$\alpha_k^2 + \beta_k^2 = \frac{k_p^2}{T^2\omega_k^2 + 1}, k = 1, 2, \tag{C.2}$$

Substituting (C.2) into (C.1) we obtain

$$\Delta_{T^2} = \frac{\delta_\beta[\delta_\beta(T^2\omega_1 + \frac{1}{\omega_1}) - 2k_p(T\cos\omega_1\tau + \frac{\sin\omega_1\tau}{\omega_1})]T_1T_\mu}{\omega_1[k_p^2(\mu^2 - 1) + \delta_\beta(2k_pT_c - \delta_\beta T_1)T_\mu]}, \tag{C.3}$$

where

$T_1 = T^2\omega_1^2 + 1$, $T_\mu = T^2\mu^2\omega_1^2 + 1$, $T_c = T\omega_1\cos\omega_1\tau + \sin\omega_1\tau$.

Then calculate the limit

$$\lim_{\omega_1\to 0}\Delta_{T^2} = \infty, \tag{C.4}$$

which shows that error Δ_{T^2} become unacceptably large when $\omega_1 \to 0$.

Appendix D. PROOF OF ASSERTION 3.4

It seen from (C.1) that quantity Δ_{T^2} is unacceptably large when $\nu = -1$. This is equivalent to the following equality, with taking into account (C.2),

$$\frac{(2\delta_\beta k_pT_c - \delta_\beta^2T_1)T_\mu}{k_p^2(\mu^2 - 1)} = -1, \quad T_c = T\omega_1\cos(\omega_1\tau) + \sin(\omega_1\tau),$$
$$T_1 = T^2\omega_1^2 + 1, \;\; T_\mu = T^2\mu^2\omega_1^2 + 1.$$

It gives

$$\delta_{\beta 1,2} = k_p\frac{T_cT_\mu \pm \sqrt{T_\mu(T_c^2T_\mu + T_1(\mu^2 - 1))}}{T_1T_\mu},$$

where $\delta_{\beta1}$ and $\delta_{\beta2}$ there are always.

Appendix E. PROOF OF ASSERTION 4.1

Consider the frequency loop transfer function of the system (1), (25), (5)

$$w_{loop}(j\omega) = \frac{k_ik_pe^{-j\omega\tau}}{(j - T\omega)\omega}. \tag{E.1}$$

In order to determine the bound of k_i, find the frequency of the ultimate point (point of intersection Nyquist curve with real axis) from the following equation

$$\text{Im}\,[w_{loop}(j\omega_u)] = \frac{k_ik_p(T\omega_u\sin\omega_u\tau - \cos\omega_u\tau}{(T^2\omega_u^2 + 1)\omega_u} = 0, \tag{E.2}$$

it is gives equation (28). Unique solution of equation (28) could be found in range (29).

The bound of k_i is determined from the inequality

$$\text{Re}\,[w_{loop}(j\omega_u)] = -\frac{k_ik_p(\sin\omega_u\tau + T\omega_u\cos\omega_u\tau)}{(T^2\omega_u^2 + 1)\omega_u} > -1, \tag{E.3}$$

it gives

$$0 < k_i < \frac{\omega_u}{k_p\sin\omega_u\tau}. \tag{E.4}$$

Denote $l \doteq \frac{\omega_u}{\sin\omega_u\tau}$ and find

$$l_m = \min_{\substack{\underline{\tau}\le\tau\le\overline{\tau} \\ \underline{T}\le T\le\overline{T}}} l, \quad \text{under condition (28).} \tag{E.5}$$

In this case expression (26) is given by (E.4).